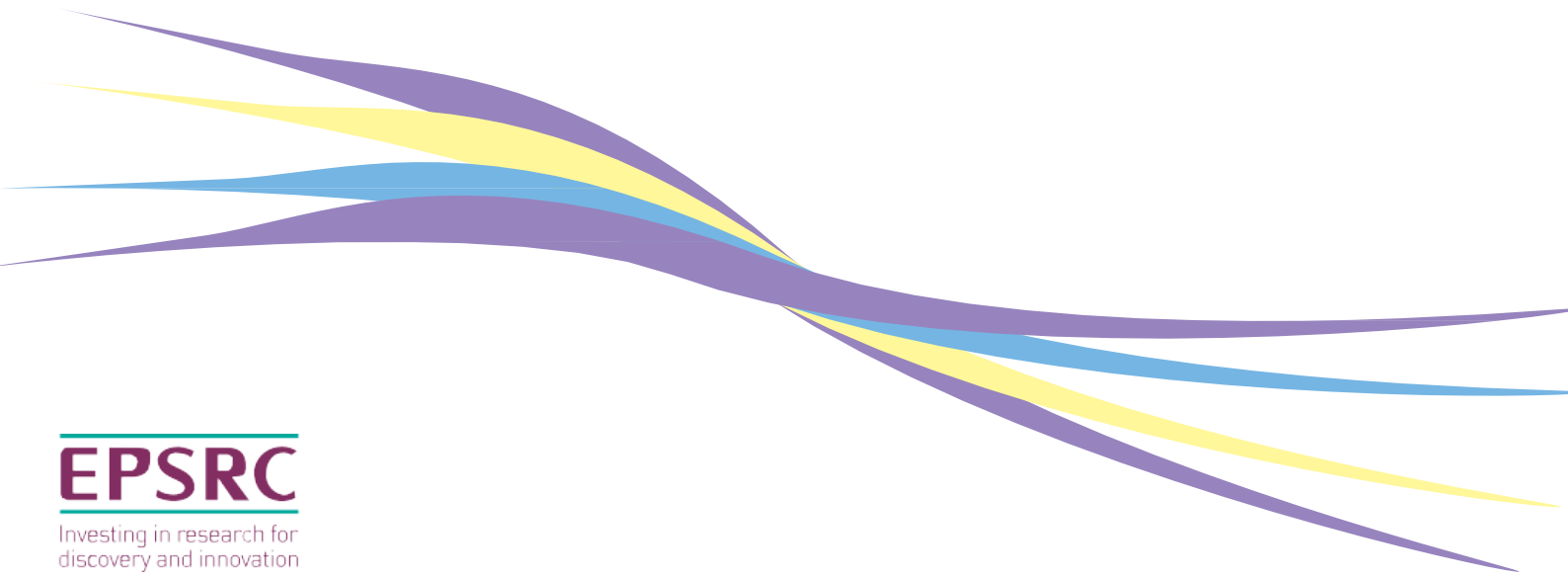
ENGINEERING AND PHYSICAL SCIENCES RESEARCH COUNCIL

PURE MATHEMATICS EVIDENCE AND ENGAGEMENT WORKSHOP REPORT

Research, discover, innovate



EPSRC
Investing in research for discovery and innovation

CONTENTS

CONTEXT

Balancing Capability is one of three strategies in our strategic plan and is an important part of managing our portfolio of investments. It aligns our portfolio to areas of UK strength and national importance. Our strategy has enabled us to champion excellence and invest in research of the highest quality in addition to securing better value for the taxpayer. Through this strategy, EPSRC supports a balanced portfolio that nurtures both discovery and challenge led research – a balance that is achieved through our engagement with business and government to influence, respond and adapt to a changing research landscape. Balancing Capability has continued to use the research area trajectories of 'grow', 'maintain' and 'reduce' and this approach will continue to enable EPSRC to balance our portfolio in line with national need.

What is EPSRC doing now as part of the Balancing Capability strategy?

EPSRC are now reviewing the research area rationales to reflect how the research base has changed over the last five years. This will enable EPSRC to align these with future aspirations for the next five years.

Why are we reviewing our rationales?

- Our original research area rationales were published in 2012 but the science landscape has continued to evolve and we need to ensure our rationales reflect this.
- We need to ensure we focus on UK strengths and nationally important areas so the UK remains internationally competitive and builds strength in areas with potential in terms of national prosperity.
- In our delivery plan, we have identified four interlinked Outcomes which collectively underpin UK prosperity: Productivity, Connectedness, Resilience and Health. We want to ensure that our research area rationales link appropriately to this outcomes framework.

The Pure Mathematics Evidence and Engagement workshop was held to build upon work EPSRC has already undertaken and to gather further evidence to support the review of our research area rationales and to provide the community with an opportunity to feed into shaping our research area strategies as we move into the next delivery plan period. The views represented in this report are a summary of points raised from the perspective of the participants at the workshop and should not be regarded as a comprehensive overview of each research area or as a comprehensive representation of the thoughts of the communities working in each research area. The evidence gathered will form part of an evidence base which will inform the development of our research area rationales in implementing the delivery plan.

OBJECTIVES

The objectives of the workshop were as follows;

- To analyse and contextualise the current UK landscape for each Research Area within the Pure Mathematics portfolio.
- To engage with the community to discuss the direction of Pure Mathematics through the next delivery plan phase.

DELEGATE SELECTION

During discussions with the Mathematical Sciences Strategic Advisory Team (SAT) it was proposed that attendees at the engagement workshop were nominated by their respective communities. To facilitate this process more than 35 heads of department at academic institutions across the breadth of the UK were contacted and were requested to submit up to three nominations for members of the academic community working in each research area to attend the workshop, with the caveat that they were unable to nominate members of their own institution. The six research areas which were being represented at the event, and required nominations were; Algebra, Combinatorics, Geometry & Topology, Logic, Mathematical Analysis and Number Theory. Representations were also made from the learned societies including the London Mathematical Sciences (LMS) and Institute for Mathematical Applications (IMA). A full list of delegates can be seen in Appendix One.

WORKSHOP PROCEEDINGS AND OUTCOMES

The workshop was divided into four sessions with the first day allocated to assessing the community perspective of the current research landscape followed by a second day focusing on future strategic directions.

EPSRC representatives from the Mathematical Sciences, Digital Economy and Balancing Capability teams were in attendance at the event to facilitate each session and to answer any questions community members wished to field.

An introductory session on the current Mathematical Sciences theme strategy and an introduction to the Balancing Capability process were provided, followed by a short presentation outlining the aims and purpose of the event. The emphasis was made that the EPSRC were there to engage and to communicate the community perspective in subsequent strategic discussions.

Participants were lead through each session in either small research area specific groups or breakout groups to cover a cross section of the pure mathematics community. Time was also permitted each day for open forum discussions to engage on topics which may not have been covered in the sessions which had been predefined by EPSRC (a 'free parking of issues' board was present at all times to record questions participants wished to address at the appropriate time).

The facilitated discussions broadly focused on the following topics:

- Analysis of each research area to identify specific areas of strength, weakness, opportunity and threats (SWOT) to the health of each research area.

- Box analysis to define current and emerging overlaps between research areas within the pure mathematics portfolio and between the applied areas of the mathematical sciences portfolio and beyond to other disciplines (e.g. Engineering, Physical Sciences, ICT)
- Identification of recent success stories as a result of pure mathematical research.
- Assessment of the current topics of opportunity and weakness to each research area and how these can be overcome.
- Identifying the relevance of Impact to Pure Mathematics and gauging the opinion of participants on how it could be measured.

The agenda for the workshop can be found in Appendix Two.

Strengths, Opportunities Weaknesses and Threats

In order to further our understanding of the current research landscape, attendees were asked to discuss what they believed the strengths, weaknesses, opportunities and threats were for their main research area.

Participants were asked to provide input for six different research areas with Logic and Combinatorics being discussed separately although they are combined within the EPSRC taxonomy. The key messages from each research area are summarised below with the complete set out of outputs shown in Appendix Three.

Algebra

- Connected very well internationally and has strong connections with other research areas within Mathematical Sciences theme and other disciplines.
- Many opportunities exist to strengthen current connections for future interdisciplinary research through algebraic geometry, number theory and theoretical computer science etc.
- Current threats to the research area include lack of systematic funding for PhD and post-doctoral research assistants (PDRAs).
- Participants identified the short term interpretation of impact as threatening fundamental research in algebra.

Combinatorics

- Strong interface with other areas of mathematical sciences as well as computer science and ICT.
- Several large groups exist which are doing high profile research, in particular those at Cambridge, Oxford, Warwick, Birmingham and Strathclyde.

- Due to a lack of research ongoing in algorithms, data structure and computational complexity from a combinatorial perspective there are less interactions occurring than could be with computer science representatives.
- In comparison to research ongoing in the EU, US and Canada a disconnection between computer science and Combinatorics exists in the areas of cryptography and structural graph theory.
- Opportunities exist to exploit emerging research interests with algebra, number theory, probability and logic as well as capitalising on grand challenges such as Big Data.
- The most pertinent weakness and threat to the UK Combinatorics community was perceived to be the difficulty in obtaining PhD funding unless the academic groups were part of a CDT.

Geometry and Topology

- A key strength to this research area is the high connectivity which exists between current research areas as emphasised by the leading topics of the minimal model programme, mirror symmetry and geometric analysis.
- Currently a lack of senior expertise exists in emerging new areas.
- Gender diversity is a strong concern for the community.
- Opportunities exist to strengthen links to other research areas such as theoretical computer science, algebraic structures, computation and computer vision as well as topological data analysis, applied topology and computational algebra.
- PhD funding to small institutions and senior academics leaving the UK were considered to be key threats to the health of the research area, as well as an over-concentration of funding geographically.

Logic

- Mathematical logic has strong links to other areas of mathematics and computer science including number theory, algebraic geometry and measurement theory.
- A number of prizes have been awarded to logicians in recent years emphasising the capability of logic within the UK.
- Current weaknesses include the lack of visibility of logic in mathematics departments with the majority of logicians present in either computer science or philosophy departments.
- There is no CDT currently in logic and a lack of undergraduate training was also highlighted as current concerns.

- Opportunities in the field of logic included exploiting links to combinatorics, topology and geometric group theory.
- Current threats included a number of recent high profile retirements and losses to the community as well as the small size of most research groups.

Mathematical Analysis

- High level of investment from EPSRC and international funders emphasises the quality of researchers.
- Symbiosis exists with other areas of Mathematical Sciences and other disciplines both nationally and internationally.
- Current capacity in the UK is low in comparison to international competitors and there exists a problem in nurturing home grown talent i.e. historic problem with lack of PhDs in analysis.
- Opportunities highlighted included growing capacity of research groups in the UK which do not currently have access to a CDT e.g. Birmingham.
- Current threats include cuts in funding to the EPSRC Doctoral Training Partnership and the loss of key leaders/emerging talent in the field to overseas research groups.

Number Theory

- The current CDT in Number Theory and Geometry was highlighted as key strength in producing PhD students.
- Current reputation and recognition of fields such as analytical number theory in the UK is a key highlight.
- Concern was emphasised over the issue of gender diversity throughout the subject at all career stages.
- Current postdoctoral fellowships are too specialised on a research focus and a system in comparison to the US based solely on merit would be better.
- Opportunities highlighted included strengthening burgeoning links to other Mathematical Sciences research areas both in the UK and abroad.
- The participants identified an overemphasis on goals and applications from their research as being a threat which could fragment the subject.

Box Analysis of Inter/Intra-disciplinary Overlap from Pure Mathematics Research Areas

Participants were asked to identify links between their current research community and adjacent areas of the Mathematical Sciences taxonomy as well as highlighting links to other disciplines. The intention of this activity was to identify areas which have emerging links or potential to link in the future whilst also considering areas where there is currently large overlap with significant potential.

This activity was interpreted slightly differently by each group participating in this exercise and attendees were uneasy confining areas which are currently linked into boxes labelled with limited potential due to the unpredictable nature of Pure Mathematics.

However, this exercise did highlight the extensive collaboration which is ongoing in Mathematical Sciences and emphasised the fluidity which exists between research areas of EPSRC taxonomy. It was clear that whilst EPSRC taxonomy is the most effective way of engaging with each research community the previously conceived boundaries between Pure and Applied mathematics are highly permeable.

Highlights of the emerging intradisciplinary links for each research area are documented below with the full documented outputs included in Appendix Three.

Algebra

- Group theory and stochastic analysis
- Lie groups in non-commutative harmonic analysis
- Numerical Analysis through tropical algebra.
- Polynomial methods in Combinatorics and incidence geometry, harmonic analysis.
- Linear algebra, spectral theory and linear preserving problems.

Combinatorics

- Mathematical Analysis through graph limits
- Probability/Statistical methods including network analysis, probabilistic algorithms and models for complex networks.
- Links through large network models are of relevance to the ATI and Microsoft research.
- Model theory
- Number theory and additive Combinatorics (Fields medals: Tao, Gowers)

Geometry and Topology

- Homotopy type theory (logic)
- Geometry and Combinatorics
- Group Theory (Fusion systems)
- Metric geometry – probability theory
- Algebraic K-Theory

Logic

- Set theory and Algebraic Topology – Current EPSRC fellow
- Model theory and Set theory links to Topological Dynamics – Work of Evans
- Logic and Combinatorics through model theory of graphs and additive Combinatorics
- Univalent Foundations/Homotopy theory/Higher Category Theory
- Model theory in Number theory (Pila and others)

Mathematical Analysis

- Inverse problems
- Stochastic Analysis and Probability
- Uncertainty Quantification
- Mathematical Biology – Collective dynamics

Number Theory

- Non-commutative geometry
- Quantum Computation
- Topological Modular Forms
- Efficient congruency in resilience

Highlights of the emerging interdisciplinary links for each research area are documented below with the full documented outputs included in Appendix Three.

Algebra

- Algebraic methods in data analysis (Persistent Cohomology)
- Computer Science

- Constraint satisfaction problems.
- Algebraic characterisation
- Statistical Mechanics – diagram algebras, correlation functions, Lie theory, representation theory.
- Theoretical Physics and Representation Theory

Combinatorics

- Computer science: Constraint satisfaction (St. Andrews) and also connections with model theory (e.g. MacPherson, Leeds).
- Algorithm Design – Structural graph theory (width parameters) (ERC goals outside UK)
- Algorithms: Computer Science – Microsoft + many top places, ERC grants.
- Confirmation theory – Error correcting codes
- Computational Complexity

Geometry and Topology

- Computer Vision
- Molecular Biology
- High energy Physics/Quantum Physics
- String Theory
- Topological Data Analysis
- Robotics – Robotic Motion and Robotic Vision
- Networks
- Cryptology (Heilbronn)
- Molecular Biology
- Machine learning and data analysis

Logic

- Formal verification of software/hardware
- Theory of programming languages
- Quantum information

- Databases and big data

Mathematical Analysis

- Imaging
- Physics – Information Theory
- Theoretical Physics
- Statistical Mechanics
- Materials Science
- Financial
- Engineering

Number Theory

- Physics
- Computer Science – Algorithmic aspects
- Additive Combinatorics
- Complexity Theory
- Cryptology – Heilbronn Funding
- Optics
- Quantum Chaos
- String Theory
- Statistical Mechanics

This session provided context for the links we know exist between the research areas of the Mathematical Sciences taxonomy and beyond. These findings are critical to highlight the importance and impact that mathematical research has on adjacent disciplines.

Previous Successes in Pure Mathematics

As Pure Mathematical research is renowned for being unpredictable and its true impact may not be elucidated for decades from its inception, a discussion was held to obtain a cross cutting perspective from the community on examples of successful research which have highlighted Pure Mathematics in recent decades.

A non-exhaustive list of the examples highlighted is listed:

- Fermat's Last theorem

- Poincare Conjecture
- Mordell Conjecture
- Classification of finite simple groups
- Mori Theory
- Influence of Theoretical Physics (Two ways)
- Work by Ben Green and Terrence Tao
- Cryptography
- Birch Swinnerton-Dyer Conjectures.
- Maths Physics Compressed sensing.
- Topological Data Analysis
- RSA discovery in GCHQ
- Andre-Oort conjecture
- Minimal Model program
- Willmore Conjecture
- The abc conjecture
- Functional analysis for medical imaging
- Donaldson work on Kähler-Einstein metrics
- Arithmetic progression of primes
- Work of Maynard, Green and Tao
- Regularity structures (stochastic analysis)
- Inverse problems – Connections to analysis
- Expander graphs – Cross disciplinary links with Computer Science

To determine how these successes were identified participants were also asked what would be the outputs of successful mathematics.

- Papers in well recognised journals
- Community networking – Sharing breakthroughs
- Invited talks
- Books

- Entering popular culture
- Attracting people in the research area
- Knowing the next question to ask
- Setting the research agenda
- Linking research to other disciplines
- Broad interest in the community
- Making Pure Mathematics accessible to other communities.
- Opens new doors/opportunities
- Unexpected connections
- A common structure which enables Pure Mathematics to translate across other areas of Mathematics
- Solving problems which are acknowledged to be ‘’hard’’ e.g. Quantum Mechanics wouldn’t have been possible without Pure Mathematics.
- ICM Speakers
- Fields Medals
- Impacts on other areas
- Citations
- Solving major problems
- Publication in top journals

This session highlighted that there are an abundance of success stories in relation to Pure Mathematics out there. The community also acknowledged that the community must play more of a role in highlighting these successes, especially in an era of shrinking funding resource to ensure the best case is put forward for pure mathematical research.

Future Strategy: Capitalising on Highlighted Opportunities and Overcoming Perceived Threats

In research area specific groups, participants were asked to assess the opportunities and threats highlighted from the SWOT session. A collation of the topics discussed and a summary of suggested actions are as follows:

Number Theory	
Topic of discussion	Community suggested action(s)
<u>Fragmentation of number theory:</u> Departments have increased in size and as a result expertise has grown in specific research topics due to the ever expanding nature of Number theory as a research area. Therefore, as the subject grows it becomes more difficult to specialise broadly and researchers tend to focus on one topic.	Promote connections between research topics and generate connections between different areas of number theory and Mathematical Sciences. Maintaining the CDT model (As used for the LGSNT) and following a similar longer length PhD model through other doctoral training schemes shall permit students the time necessary to learn the research area more broadly.
<u>Diversity in Number Theory/Mathematical Sciences:</u> Gender balance was highlighted as a significant weakness in Number theory (and in all research areas of Pure Mathematics)	Improving flexibility in the grants process to accommodate career breaks.
<u>EPSRC Funding:</u> Over-emphasis on funding the project rather than the candidate.	• Fund the candidate rather than the project, by providing the correct candidate with funding then they will develop and use the money most efficiently. • Horizon 2020 programme is a large source of funding. Community should work together to seize opportunities such as new calls within the framework.
<u>Impact:</u> The word impact is overloaded and does not feel relevant to Pure Mathematical research.	Re-write the pathways to impact guidance document as it is currently too generic and confusing in regards to what can be asked for and what cannot.
<u>Big Data:</u> Opportunity to work with the themes of the Alan Turing institute to show that number theory can contribute.	Number theory could have a significant influence through formulating abstract theory, understanding algebraic statistics, generation of new subject topics. Links could also be generated between number theory and physics to analyse large swathes of data.

Logic	
Topic of discussion	Community suggested action(s)
Strong interconnections exist between Logic and Computer Science. Maintaining these links is imperative as many recent advances have occurred because of these links.	A multi-city CDT in Logic and related research areas to train students with broad backgrounds.
Applications of logic to other areas of Mathematics. Aim is to raise awareness of Logic in the Mathematical Sciences community. Has the potential for cross-fertilisation of ideas across research area boundaries.	Expand communication of central issues in mathematical logic to other mathematicians.
Current threat to the health of the logic research area is the vulnerability to small research groups due to retirement/departures. This is very pertinent as few logic groups in logic in the UK.	More independent post-doctoral fellowships to help young researchers.

Combinatorics	
Topic of discussion	Community suggested action(s)
Emerging areas which Combinatorics can play a role.	• Mathematical Biology • New Methods in statistical mechanics (and statistics more generally)
Growing smaller research groups.	• Better access to PhD students • Improve accessibility of Fellowships in relevant research areas.
Interface with other research areas.	• Scheme to encourage two PIs from different areas • Devise and incentivise grants in between these areas as it is currently safer to stay in own research areas.

	• Location of where the university and person are based means that in a lot of cases there are not a lot of people to collaborate with. • A benefit to Combinatorics would be working in the interface with other research areas and providing them with the methods to be incorporated into their discipline. • Must catch up with the current interdisciplinary research in the US. • Currently there are less groups working between MS and CS to act as translators between the two disciplines in comparison to the number of people working in these areas in the US. (An example of a good collaboration would be the St. Andrews and Warwick collaboration which has been a success yet to be replicated again in the UK)
Competition with computer science movement in North America.	Out of our control.

Geometry and Topology	
Topic of discussion	Community suggested action(s)

Over concentration of research funding:
• CDTs need to allow for smaller cohorts to increase specialisms in research area specific topics
• Funding for workshops in isolated locations/subject topics.
• Programme grants concentrate funding but where is the added value? Are resources being used efficiently?
	• Calls for regional funding/development
	• Structure to look after the health of the discipline
	  ○ Look at the breadth of the discipline and analyse using statistics.
	  ○ This could be organised by the LMS or CMS but requires money and political organisation.

• Students need researchers and vice versa, concentration of research does not work without concentrating students and the infrastructure of academia does not currently allow this.	• Smaller programme grants with more inherent flexibility for PDRAs. • Spread out research excellence – shall benefit the economy • Create a richer eco-system of maths to allow intradisciplinary research to work.
Applied Topology	Joint institutional programme grant application in applied topology or a network grant to support lots of small opportunities together.
Poor gender diversity in Maths.	• Female leaders in the field could design topics for collaborative work • Applications to work on these projects could be selected by a panel of leading experts ○ Benefits to this include networking and training young researchers. • Provide role models for younger generation. • Hold workshops with academic papers being considered as significant outputs.

Mathematical Analysis	
Topic of discussion	Community suggested action(s)
Britain's potential exit from the European Union would make European Research Council funding inaccessible.	EPSRC must consider this scenario
Cuts in the doctoral training partnership funding	Various areas will have no trained students and as such the health of the discipline will suffer. Therefore, there needs to be a balance between DTP and CDT funding.
Retention of leaders/Potential leaders: Mid-career researchers made lots of offers from abroad and creates a fragile system. These areas are having a negative impact on other research areas.	Protected time for research will create a productive research environment. Continued support from the EPSRC for analysis is a big factor in the retention of people.

PhD training is too short in the UK and puts students at an unfair disadvantage.	
Growth in the area across a wide front.	Natural evolution of Mathematical analysis, EPSRC could help this growth through career integration grants.

All	
Topic of discussion	Community suggested action(s)
How can Mathematical Sciences work better with industry?	• More formal recognition of consultation between industrial partners and the academic community. • Increased contact with past students. • Holding workshops for industrial partners to highlight problems to mathematicians. • Increase the visibility of Pure Mathematical research by publicising examples and case studies.
How to improve E&D in Mathematical Sciences.	• Think about E&D when inviting speakers • Post-doc experience can currently go against women (can be too long) • More awareness of unconscious bias. • UK doing ok in comparison to some/several other countries • Extending fellowships to cover maternity and paternity leave good but this should also be done for PhD students as well. • Real issues with percentage of women being lost through the pipeline ○ Need real data to monitor where

	they go. • Female role models are necessary to act as mentors. • Athena Swan – Not primarily focused on reaching certain diversity levels but raising awareness of the diversity issues which exist is more important. • Culture Change – Aim for standard E&D practise

Impact: What is Impact and How Could it be Measured?

The RCUK definition of impact considers accounts for research having academic, economic or societal beneficiaries. This definition is much broader than that for the Research Excellence Framework (REF) which excludes impact for academic beneficiaries.

In light of this RCUK established the Pathways to Impact document across all grant proposals in order to identify a clearly thought through route to establishing the impact of funded research. Whilst establishing a transparent pathway to impact for research in fundamental disciplines can seem challenging, the broad definition of impact means that all funded research can provide tangible benefits by contextualising who the beneficiaries are of the research.

Ahead of this session EPSRC provided a presentation on the monitoring of research outputs/impact through Researchfish and the extensive list of activities which can be included in the Pathways to impact document.

Attendees were then placed in their breakout groups to discuss from their perspective what impact is and to discuss its importance. This was followed by a second activity to understand the methods by which attendees currently measure the impact of their own research and for them to suggest methods by which impact could be measured in the future by EPSRC.

What is Impact and why is it important?

An open and robust discussion was had within the various breakout groups. A list of the thoughts gathered is included.

What is impact?

- Progress and interest by our colleagues – research quality.

- Links with other academic areas within Maths or through bridging to other disciplines.
- Building ‘’tools’’ – Building intellectual capital/infrastructure.
- Creation of new technologies such as magnetic image scanners.
- Big theorems.
- Significant effect on future direction/activity in relevant areas.
- Permeating mathematical styles of thinking beyond mathematics
- Examples; Google (Page Rank algorithm), Error-correcting codes and other applications in communication technology.
- Engagement – Public engagement and Engagement in Mathematical Sciences
- In hindsight – What are the chain of events for big discoveries – where does maths play its part?
- Conferences – Lecture courses/workshops, Key note speaker invitations.
- MathsSciNet Citations
- Other people reading your papers
- Being invited to conferences
- Research worthy of being discussed by public engagement experts such as Marcus du Sautoy.

Why is impact important?

- Inspiring people.
- Impact within Maths has ‘ripple effect’.
- Generates wider public interest.
- Educates a wider audience.
- Justification of resources.
- Areas of research with no impact beyond themselves wither and die.
- We have a culture of measuring things in comparison to other nations.
- Necessary to prevent ideas from languishing
- Necessary to justify use of public funds
- Stimulate interest in new generations

- Enhances Teaching
- Improves self esteem
- Changes opinions
- Inspires new generation
- Informs policy – Results in better decisions
- Legacy of your research.
- Demonstrates that Maths is important.
- Change research knowledge in field
- Develops other people
- Allows Maths to be the infrastructure on which things can be drawn – Enabling
- Need to show failures to stop some work being repeated
- To get EPSRC money
- People impact – Students/academics want to work with you
- Furthering career opportunities
- Developing other people
- Changes research knowledge in another field
- Someone knows who you are and what you are doing
- People adjust their research due to the impact of others work
- Intellectual and social satisfaction
- Impact is a change in momentum
- Role model for what's possible.
- Training – Need for analytical and critical thought
- Turning people into research mathematicians
- Under graduate training need to be done by researchers otherwise they are learning a 30 year old subject.
- Necessary – fulfils a need to communicate
- Other people use your work
- Often very long term

- Necessary for progress in science, society and the economy
- New Knowledge
- To keep us out of the dark ages

The general consensus was that whilst some of the academic participants disagreed with the notion of their research being required to impactful, they understood that from an EPSRC perspective it was critical to highlight the impact of their research to permit the best case for support to be made for mathematical sciences research. This list also highlights that research impact creates a legacy for future generations of researchers.

How should (or could) impact be measured?

- Invitations to speak – Conferences
- ResearchFish is too fragmented, final reports for the area, contextualised narrative.
- Books
- Destination of PhD students
- Number of PhD Students
- Collaborations – looking at the impact of the research you use in your research.
- Having others prove my conjectures
- Publishing papers
- Giving lectures.
- Need more expert committees
- Summary paragraph describing highlights of what's been achieved in the grant written for non-experts.
- Publications
- Invites to conferences
- National Prizes
- Next destination/ Career projection at post-docs
- Awards
- Grants
- Economic – Through new technologies
- Social – Study of collective dynamics lead to better design of public spaces, reduction of crime (through interaction with police departments)

- Academic – citations, professional judgement and ICM invitations.
- Quality of journal in which research is published
- Ask experts about likelihood of impact
- Invitations to speak
- Timescale for impact can be very long, therefore, citation counts are very unreliable.
- MathSciNet
- Many forms of impact are essentially not reducible to numerical measure
- Measures of centrality
- Citation tree growth
- All numerical measures are likely to be unreliable in the short term.
- Mathematics is genuinely outliers in terms of the long-time scales to adequately measure impact of any kind.

A range of suggestions were made for the calibration of impact from Mathematical Sciences research. The methods suggested will be considered and feed into EPSRC discussions on impact within the Mathematical Sciences theme and in relation to the EPSRC strategic aim of accelerating impact.

SUMMARY OF ISSUES RAISED

The workshop incorporated two discussion sessions which covered a range of topics. A board of issues was also placed at the front of the room to generate further discussion topics. A summary of the issues raised are discussed below.

- Mathematics is a fast changing environment and currently our style does not account for this. Academic departments won't get grants if leading academics do not push their area.
- Balancing capability was discussed at the last EPRSRC and Council for Mathematical Sciences (CMS) bilateral meeting where EPSRC Chief Executive Phil Nelson suggested that the community make the case for supporting each area during the balancing process to share the balancing burden.
- Broad support for more fellowships but the need for increasing support through this route needs to be articulated to EPSRC.
- CDTs are they working for the Mathematics community.

- DTP funding cut to Mathematical Sciences is a significant concern to the community and in the near future CMS plan to survey academic institutions to find out how the DTP allocation was awarded to Mathematical Sciences departments.
- Acknowledged the next few months is important for getting the best settlement for Mathematical Sciences and translating that into plans for the next few years.
- Community are wary of too much immediate change to the current landscape.
- The absence of representation for probability theory and its role in Pure mathematical research was highlighted.

NEXT STEPS

This event was seen as a platform to build upon our engagement with the Pure Mathematics and wider Mathematical Sciences community in order to ensure the communities feel represented with a balanced point of view in any strategic discussions going forward. Throughout 2016 and beyond EPSRC will be continuing to main a constructive dialogue with the research community, especially during the ongoing Balancing capability process.

i) Since the event EPSRC has held a subsequent workshop with the Applied Mathematics community and a short summary report of this workshop shall also be published.
ii) There are following events to engage the community perspective on Mathematical Physics and Statistics and Probability being planned to take place within the next 12 months. (A recent event was held with the Operational Research community and a report for this event will be published in due course).
iii) This report will be considered by the Mathematical Sciences SAT prior to publication and dissemination to the rest of the community. If the SAT highlights possible actions as a result of this report then they will be considered in due course.
iv) The outputs from the workshop will be incorporated into the evidence gathering stream for the Balancing capability process to ensure a community perspective remains a key focal point during strategic discussions.
v) The community generated Landscape snapshot documents shall also be published on the EPSRC website to provide the community perspective of how the research landscape has changed during the last 5-6 years.
vi) Issues and suggestions raised in regards to equality and diversity issues and support for early career researchers will be fed into ongoing strategic actions plans for each of these

topics, which remain to be key foci for the EPSRC throughout the next delivery plan phase.

vii) As reiterated at the event this is one part of a developing engagement programme with the Mathematical Sciences community and numerous opportunities shall arise during the next 12 month period for interactions between EPSRC and the research community.

FEEDBACK

Details of the feedback received can be viewed in Appendix Four.

APPENDIX ONE – DELEGATE LIST

Jonathan Bennett	University of Birmingham
Tara Brendle	LMS
Martin Bridson	University of Oxford
Ken Brown	University of Glasgow
Kevin Buzzard	Imperial College London
Peter Cameron	University of St. Andrews
Tony Carbery	University of Edinburgh
John Cremona	University of Warwick
Mirna Dzamonja	University of East Anglia
Nicola Gambino	University of Leeds
Iain Gordon	University of Edinburgh
Andrew Granville	University College London
Ben Green	University of Oxford
John Greenlees	University of Sheffield
Memhet Haluk Sengun	University of Sheffield
James Hirschfeld	University of Sussex
Jon Keating	Heilbronn Institute
Daniel Kral	University of Warwick
Daniela Kuhn	University of Birmingham
Assaf Libman	University of Aberdeen
Ying-Fen Lin	Queen's University Belfast
Dugald MacPherson	University of Leeds
Robert Marsh	University of Leeds
Marta Mazzocco	IMA
John McKay	University of Bristol
Graham Niblo	University of Southampton
Michela Ottobre	Heriot Watt University
Beatrice Pelloni	University of Reading
Richard Pinch	GCHQ
Mike Prest	University of Manchester
Michael Ruzansky	Imperial College London
Gregory Sankaran	University of Bath
Eugene Shargorodsky	Kings College London
Alexander Stasinski	Durham University
Peter Symmonds	University of Manchester
Jack Thorne	University of Cambridge
Ulrike Tillman	University of Oxford
Peter Topping	University of Warwick
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APPENDIX TWO – AGENDA

Engineering and Physical Sciences Research Council

Mathematical Sciences Theme: Pure Mathematics Engagement Workshop

28 and 29 January 2016 Hilton Metropole Birmingham NEC

28 January – Day 1: Understanding the research landscape

10.30 – 11 am	Registration (Tea/Coffee)

Introductory Session

11 – 11.05 am	Welcome and Introduction
11.05 – 11.20 am	EPSRC Mathematical Sciences Update
11.25 – 11.40 am	EPSRC: Balancing Capability
11.40 – 11.55 am	Icebreaker

Session 1

11.55 – 12 pm	Introduction to the session
12 – 12.30 pm	Information Bursts – Landscape Document authors
	Tea/Coffee (In the room)
12.30 – 1.30 pm	Analysis of individual Research Areas
1.30 – 2 pm	Group discussion.
2 pm – 2.45 pm	**Buffet Lunch**

Session 2

2.45 – 2.50 pm	Introduction to session
2.50 – 3.30 pm	Analysis of Inter/Intra-disciplinary overlaps (Part I)
3.30 – 3.45 pm	Tea/Coffee
3.45 – 4.30 pm	Analysis of Inter/Intra-disciplinary overlaps (Part II)
4.30 – 5.15 pm	Open Discussion
5.15 – 5.30 pm	Maths Team Debrief
7.00 pm	**Dinner**

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29 January – Day 2 – Pure Mathematics: Looking forward

Introductory session 2

9 – 9.15 am	**Tea and Coffee**
9.15 – 9.20 am	Overview of Day 2
9.20 – 9.30 am	**Energiser**

Session 3

9.30 – 9.35 am	Introduction to the session
9.35 – 10 am	Pure Mathematics Milestones/Success Stories.
10 – 10.15 am	Refresh: Current Landscape analysis
10.15 – 10.45 am	Pure Maths: The next 25 – 50 years (Part I)
10.45 – 11 am	**Tea/Coffee**
11 – 12 pm	Pure Maths: The next 25 – 50 years (Part II)
12 – 12.45 pm	**Buffet Lunch**

Session 4

12.45 – 12.50 pm	Introduction to this session
12.50 – 2 pm	Pure Mathematics and Impact

Session 5

2 – 3 pm	Open Discussion

Goodbye

3 – 3.30 pm	EPSRC Round Up of the Event

APPENDIX THREE - OUTPUTS

Strengths, Opportunities, Weaknesses, Threats

Research Area: Algebra

Strengths

- International Connectivity
- Numbers of regional networks with International outlooks
- Connections with areas outside Mathematics
- Expertise spread across the UK
- Strong relationship with larger parts of other mathematical disciplines
- Students compete well internationally i.e. PDRAs, PhDs.

Weaknesses

- High workload = isolation of some groups
- Absence of systematic funding (PhD & Postdoc)
- Diversity of research community

Opportunities

- UK having world leading centres to attract leading International researchers
- Continuing presence of algebra at key world centre. UK connections to these leading activities.
- Systematic long-term support of mathematics infrastructure (i.e. INI and ICMS)
- Possibility to attract top world stars into a fertile environment.
- Many opportunities for future interdisciplinary research: algebraic geometry, number theory, theoretical computer science etc.

Threats

- Career Progression problems (i.e. 'People Pipeline')
- Hiring potential internationally limited by fears about impact i.e. REF

- Narrow short term interpretation of impact threatens fundamental research in algebra.
- Need for many talented young people (PhD + Postdocs) to go abroad.
- CDTs

Research Area: Mathematical Analysis

Strengths

- Quality of researchers – Evidenced through investment by EPSRC and other funders
- S and I awards stimulated growth in this area.
- UK is an attractive place to do analysis.
- Well integrated with other areas of mathematics.
- Internationality – Strong collaborations, outward focus and research exchange.
- Symbiosis with other areas of Maths and other disciplines.

Weaknesses

- Too small in comparison to international competitors i.e. lower proportion of number of researchers in the UK.
- Not enough home grown talent i.e. historic problems with a lack of PhDs in analysis.
- PhD training is too short in comparison to USA and Europe.
- Analysis is not taught in enough depth at undergraduate level.

Opportunities

- Increased interaction with other areas (probability, big data, statistics, applied maths, geometry, physics, materials science etc.)
- Growth in the area.
- Opportunity to grow research groups in the UK which don't have CDTs e.g. Birmingham.

Threats

- Cuts in DTP funding
- Key leaders in the field are moving abroad, UK producing successful analysts but they are leaving.
- Retention of leaders/potential leaders.

- Money ends up too thinly spread.
- Because analysis is very broad it is difficult to select appropriate reviewers.
- In USA positions in analysis are being given to people who are not developing analysis.

Research Area: Geometry and Topology

Strengths

- Strong international connections
- Excellence
- Links with Maths Physics
- Minimal modal programme, mirror symmetry, manifolds field theory, geometric analysis.
- Strong community - Connectivity between researchers.

Weaknesses

- Lack of senior expertise in new areas
- High eligibility threshold for programme grants
- CDT mechanism is flawed and it disadvantages smaller institutions and favours the biggest institutions.
- Inflexible HR practise.
- Poor gender balance
- Concentration of PhD supervision.

Opportunities

- Increasingly central and flexible so has been able to make broad connections within and outside Mathematical Sciences.
- Links to areas such as theoretical computer science and algebraic structures, computation and computer vision. TDA, applied topology, computational algebra.
- Women in Topology, Banff – Ran by academics and results in a buyout of teaching.
- At Banff there is ample opportunity for networking and meeting with senior role models.
- Enables attendees to work on projects in advance and come together for a week to collaborate, network etc.

Threats

- Senior academics leaving the UK – Donaldson as an example.
- PhD funding for small places i.e. over concentration of funding
- Immigration policies.
- Poor gender diversity
- Reduced number of something in topology
- Reduction in accessible grant funding.

Research Area: Logic

Strengths

- Many areas of logic have connections to other areas of mathematics and to computer science (Results in number theory, algebra, algebraic geometry and measurement theory)
- Good interconnectivity between different areas of logic.
- 4 INI programs in recent years.
- Major achievements in recent years.

Weaknesses

- Relatively small number of people within mathematical logic (many more in CS-logic)
- No CDT
- Lack of undergraduate training.

Opportunities

- Univalent foundations/Homotopy type theory and impact on computer proof checking.
- Links to Combinatorics, topology, geometric group theory.
- Build on interconnectivity.
- Attract EU/ERC funding.
- New approaches to training (CDT, links)

Threats

- Retirements
- Small size of some groups
- Few places to offer opportunities of logic appointments.

Research Area: Combinatorics

Strengths

- Additive Combinatorics
- Extremal Combinatorics
- Probabilistic Combinatorics
- Easy to attract strong people and easy for former students to find jobs in the real world.
- Several groups with critical mass and several places close to critical mass, in particular Cambridge, Oxford, Warwick, Birmingham and Strathclyde.
- Interface to other areas (Inside and Outside of Maths – CS, OR)

Weaknesses

- Small amount of research in algorithms, data structure and computational complexity in comparison to CS – Less opportunities for interactions.
- Areas of disconnect between CS and Combinatorics – cryptography, structural graph theory (Compared to rest of EU, US and Canada)
- Hard to obtain PhD funding unless a part of a CDT.

Opportunities

- Funding for places with some strong people to achieve 'critical mass'
- Interface to other areas:
    - Inter – CS, Big Data and OR
    - Intra – Algebra, Number Theory, Probability and Logic

Threats

- Senior academics leaving the UK.
- Competition with the US in areas of the interface with CS
- Competition for PhD graduates with industrial companies (Google, GDRS)
- Dependence on EU funding
- Many UK Mathematicians are not from the UK – less familiarity with the system.

Research Area: Number Theory

Strengths

- Good age distribution
- LSGNT CDT
- Smart people proving great theorems
- Wide diversity of topics
- Very good PhD students from both UK and outside
- Many people working on ‘’hot topics” at a high level
- Many universities in Southern England have a decent number of people
- Level of recognition in analytical number theory is particularly high. (ICM speakers)

Weaknesses

- Low proportion of women
- Difficulty retaining/attracting PDRAs – insufficient structure
- Too many postdoctoral fellowships have too specialised a focus unlike in the US were they are strictly merit based funding.
- Imbalance in PhD funding.
- Short length of PhDs
- Lack of flexibility for PhD level researchers – moving to different areas/supervisors.

Opportunities

- Number theory is rich in links to other Mathematical areas.
- Participation in more European programmes
- More interaction with people in Paris

Threats

- Overemphasis on applications and impact
- Fragmentation of the subject
- Increasing depth in some areas it take years to get to the forefront of research
- Over emphasis on goals
- British exit from the EU.

Interdisciplinary Overlaps between Mathematical Sciences Research Areas and Other EPSRC Themes

Research Area: Algebra	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Tropical Algebra e.g. Financial Mathematics and Biology – Genetic Trees Quantum/Categorical Algebra + Compute languages Financial Mathematics e.g. Tropical algebra and Hopf algebra. Real world cryptography and Algebra. Algebra and Theoretical Computer Science: Coding theory and cryptography. Symmetry and viruses e.g. Twarock at York & Durham. Robotics (Configuration spaces, fundamental groups)	Algebraic methods in data analysis (Persistent Cohomology) Computer Science Constraint Satisfied problems. Algebraic characterisation Statistical Mechanics – diagram algebras, correlation functions, Lie theory, representation theory. Theoretical Physics and Representation Theory

Research Area: Geometry and Topology	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Contextually in Computer Science Bioinformatics Mathematical Finance Modelling in social sciences (i.e. relationships mapping/random complexes) Proteomics Topological Codes – Quantum Computing Meta materials – developing new materials, domain walls Neuroscience + Algebraic Topology	Computer Vision Molecular Biology High energy Physics/Quantum Physics String Theory Topological Data Analysis Robotics – Robotic Motion and Robotic Vision Networks Cryptology (Heilbronn) Molecular Biology Machine learning and data analysis

Research Area: Number Theory	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Medical Imaging?	Physics Computer science – Algorithmic aspects ▪ Additive Combinatorics ▪ Complexity Theory Cryptology – Heilbronn funding Optics Quantum Chaos String Theory Statistical Mechanics

Research Area: Combinatorics	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Big Data – Network analysis Sublinear algorithms Testing Computer Science: Large Databases for mathematics (e.g. finite groups, L-Functions – John Cremona programme grant) Computer Science: Network coding, memoryless computation (Riis, CS, QMUL) Property testing – Evidence: Invited talks – ICM, FOCS, STOC, ERC Regularity method – Combinatorial limits (ERC grant – Shapira)	Computer science: Constraint satisfaction (St. Andrews minion) and also connections with model theory (e.g. MacPherson, Leeds). Algorithm Design – Structural graph theory (width parameters) (ERC goals outside UK) Algorithms: Computer Science – Microsoft + many top places, ERC grants. Confirmation theory – Error correcting codes Computational Complexity

Research Area: Logic	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Philosophy/Foundations Computer game theory and economics Algorithms	Formed verification of software/hardware Theory of programming language Quantum information Databases and big data

Research Area: Mathematical Analysis	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Chemistry – Molecular Dynamics Data Science and Stats – Alan Turing Institute	Imaging Physics – Information Theory - Theoretical Physics - Statistical Mechanics Materials Science Financial Engineering

Intradisciplinary Overlaps between Research Areas within Mathematical Sciences

Research Area: Algebra	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Group Theory & Stochastic Analysis Lie groups in non-commutative Harmonic analysis Numerical Analysis (Tropical Algebra) Polynomial Method (in Combinatorics) and incidence geometry, harmonic analysis Linear Algebra, spectral theory and linear preserving problems.	Group Theory and anabelian geometry/arithmetic geometry. (Profinite groups) Algebraic Geometry and non-commutative algebra. Algebraic topology Model theory and algebra Hyperbolic geometry, group theory and cluster algebras Linear numerical algebra and representation theory/algebraic groups.

	Representation theory and categories. Algebraic geometry and cluster varieties. Algebraic Combinatorics Algebra and spectral theory Algebraic geometry (commutative algebra) p-adic representation theory and number theory. K-theory in classification of C*-Algebras Geometric group theory. MSRI special year '16.

Research Area: Geometry and Topology	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Commutative Algebra Group Theory (Fusion Systems) Spaces with Ricci bonds Geometry and Combinatorics Homotopy type theory (Logic) Algebraic K-Theory Stochastic Analysis (in manifolds) Metric Geometry – probability theory - Analysis - Optimal Transport	Model Theory +Algebraic geometry Statistical/Random topology Algorithms Categorification/Representation theory/knot theory Coarse geometry Combinatorics Non-Commutative Logic (Topos theory) Integrable systems (Maths Physics) Algorithms Algebra – Representation Theory - Derived Categories Statistics (Slope Theory) Algebraic Number theory (arithmetic geometry)

	Continuum Mechanics Analysis – C* Algebras - Geometric Function theory - Geometric flows

Research Area: Combinatorics	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Analytic approximations of large discrete structures – Microsoft Research Analysis – Graph Limits Probability/Stats – Methods - Network Analysis - Prob. Algorithms - Models for complex networks Networks Large network models (Probability, stats, analysis) – ATI, Microsoft Research Model Theory Regularity Method – Number Theory Number Theory – Additive Combinatorics (Fields medals: Tao, Gould) Harmonic Analysis/Geometric Measure Theory Diagram Algebras PDEs	Algebra – Transformation semigroups (esp. synchronization) (Groups at Hull and St. Andrews) Finite Geometry and Coding theory Algebra – Expanding group theory Probabilistic methods – Work done by Microsoft research Representation Theory – Gordon Edinburgh Cluster Algebra Theory (Fellowships at Leeds) Polytopes (algebraic geometry) Integrable Systems (Random matric models) Mirror Symmetry Tropical Geometry Finite group Theory

Research Area: Mathematical Analysis	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Inverse Problems (limited current overlap in UK) – Applications to	Maths analysis – Differential Geometry - Combinatorial Geometry

engineering and development of theory. Probability + Stochastic Analysis Uncertainty Quantification Maths Biology – Collective Dynamics	- Algebraic Geometry - Relativity - Topology Probability and Stochastic Analysis Number Theory Computational Analysis Numerical Analysis

Research Area: Logic	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Set Theory & Algebraic Topology/Category Theory – EPSRC Fellowship A. Brooke-Taylor Model Theory & Set Theory in Topological Dynamics – Evans in UK with potential links to research elsewhere. Univalent Foundations/Homotopy Theory/Higher Category Theory – International activity, impact on several disciplines & computer proof dreading. Logic and Combinatorics (Proof theory of graphs, additive Combinatorics) – Dzamonja, Review Maths of Rathigen.	Model Theory & Number Theory/Geometry – Recent results of Pila & co-authors - Work of Zilber and Wilkie Model Theory & Modules – Potential apps to geometric group theory Model Theory & Group Theory – Well established in the UK Set Theory & Measure Theory – Work ongoing in UK and potential for overseas collaborations.

Research Area: Number Theory	
Limited Current Overlap with Potential for future overlap	Substantial current overlap with scope for increase overlap
Non-commutative geometry Quantum Computation Topological Modular Forms Efficient congruency in resilience	Probability theory Computation – Programme Grant Representation Theory Algebraic Geometry – LGNST CDT.

	p-adic analysis Logic – Pila and Wilkie Ergodic Theory Combinatorics Mathematical Physics Random matric theory & algebraic k-theory -

APPENDIX FOUR - FEEDBACK

Response Rate

The overall response rate from delegates was 60%.

Overall Impressions

Question: How did you feel about each session of the workshop?