Mathematical Challenges in the Forecasting of Environmental Change: Mitigation and Adaptation Approaches

Workshop Report

30th September – 1st October 2013

Hilton Newport Hotel, Newport

EPSRC
Pioneering research and skills

Living With Environmental Change

Background

In the year of Mathematics of Planet Earth, the EPSRC wished to explore how UK expertise in the mathematical sciences can contribute to the challenges within Living with Environmental Change (LWEC). The effects of environmental change are wide ranging and this is reflected in the scope of LWEC; areas of interest include resources, civil infrastructure, climate, health, ecosystems and societal challenges.

The workshop was jointly organised by the EPSRC Mathematical Sciences and LWEC themes. Participants were sought through a call for Expressions of Interest and included mathematical sciences researchers and researchers working within various areas of LWEC. A full list of participants is available in appendix 1.

The aim of the workshop was to identify and prioritise the opportunities where mathematical sciences research could bring new insights and add significant benefits to the forecasting, mitigation and adaptation to environmental change and establish the shape of a future funding activity. The focus on forecasting at the workshop had been informed by a round table meeting with key researchers earlier in the year.

Format of the workshop

The workshop began with a presentation introducing the aims of the event. Following this there was a networking session; participants were split into groups and asked to introduce themselves to the rest of the group, the groupings were changed at intervals so that participants could meet as many people as possible in the time available.

The workshop was then divided into the following facilitated group discussions, the outputs of which are summarised later in this report. The members of the groups were changed for each session to facilitate networking and broader discussion.

1. What does 'forecasting' mean to you?
2. Challenges in environmental change
3. How mathematical sciences can contribute to the challenges in environmental change
4. Connections between the challenges and mathematical sciences identified.

The following participants gave presentations during the first half of the workshop as examples of where mathematical sciences might bring new insight to LWEC challenges:

- Professor Gareth Pender, School of the Built Environment, Heriot-Watt University
- Professor Margaret Bell CBE, Transport Operations Research Group, Newcastle University, "Mathematical Challenges in Delivering Carbon and Air Quality Targets"
- Professor Tim Palmer, Department of Physics, University of Oxford, "Mathematical Challenges in Forecasting Climate".

A 'thoughts' board was provided throughout the workshop to allow participants to highlight anything they felt was not being captured through the main sessions. In recognition that the participants only represented a small part of the community an 'additional expertise' board was also provided so that it could be noted which areas of expertise could contribute to the discussion but were not present.

Outputs

1. Forecasting

This was a 'world-café' style discussion, with ideas recorded informally on tablecloths. The participants were split into 6 groups and each group was asked to capture their thoughts to the following questions:

- What is Forecasting?
- What does it mean to you in terms of research/expertise?
- What does it mean for industry?
- What stakeholders should be involved? (outside industry)
- Strengths? Weaknesses? Threats? Opportunities?
- What does it mean outside the UK?
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The discussions on Forecasting were wide ranging, but the following were common themes across the six groups.

1. Forecast Models – which can be used for a variety of applications and built though a variety of methods;
2. Uncertainty – risk, quantification of uncertainty, verification;
3. Scales - including multi-scales in models, between micro and macro and over timescales;
4. Prediction/Projection – forecasting can do these things for many application areas;
5. Data for Forecasting – availability, data correctness, data validity, amount of data etc.;
6. Forecasting for Climate/Weather/Environmental Change – the most common application area discussed and raised;
7. Communication of forecasting – outcomes, predictions, reliability and how to get people to understand/appreciate/believe what comes from forecasting;
8. Forecasting in social science – including behaviours and social impacts.

The forecasting session fed heavily into the subsequent sessions, with some of the key themes mentioned above playing a strong role in the remainder of the workshop. The participant's notes can be found in appendix 2.

2. Challenges in environmental change

This session was a 'world-café' style discussion, with ideas recorded informally on post-it notes. The participants were asked to discuss in groups what they considered to be the challenges in environmental change. Groups were asked to cluster their points and summarise these clusters. The summaries from each table were bought together, clustered into high-level challenges, and given titles. Each participant was asked to identify which two high-level challenges were most important, resulting in the following:

1. **Uncertainty and variability:** Areas highlighted by the participants under this heading centred on improving the ability to quantify uncertainty and variability within and

between models and its subsequent representation/communication, the type of data required and assessing the value of different types of data.

2. **Integrating models/multi-scale models:** Under this heading the participants highlighted the potential of modular/flexible models that could be linked to create multi-scale models. The ability to integrate models could allow the boundaries and interactions between different environmental challenges to be probed. However, it would then be important to trace the assumptions and uncertainty between the linked models.

3. **Decision support and communication:** A main focus raised under this heading is the communication of uncertainty when it can mean different things to different people. It was suggested that the communication tool and method should be tailored to target a specific audience. When making outputs relevant to decision making it was recognised that temporal and spatial scales were important as was the clear provision of uncertainties in the information provided.

4. **Extreme/high impact events:** Within this area points raised included: the effect of tipping points and the result of interventions. How to model interventions, when such models would likely require components that are currently unknown was raised. How long it takes the system to reach equilibrium following a tipping point/intervention is also important to consider.

Other high-level challenges that came out of this session included;

5. Behaviour
6. Population change

=7. Resource Management
=7. Only one real world
=7. Sources of Pollution

More detailed notes of the session are available in appendix 3.

3. Mathematical sciences that can contribute to the challenges in environmental change

Participants discussed in groups, world-café style, how mathematical sciences might be able to contribute to addressing the challenges identified in the previous session. Two groups were assigned to each of the top three environmental challenges, after a period of time participants were then encouraged to read and comment on what had been noted by other groups.

A small sample of the points discussed are listed below, more detailed notes of the session are available in appendix 4.

- **Uncertainty and variability:** Issues about the availability of data; propagation of uncertainty along the model chain; extremes versus average; integration of scales
- **Integrating models/multiscale models**: designing models so that they can fit together e.g. data formats, time and spatial variables, choosing the right model variables, propagating uncertainty, diversity of models; enabling two-way feedbacks between each component of an integrated model; model validation.

- **Decision support and communication**: well posed options/questions; optimisation methods and dynamics; two-way communication between researcher and stakeholder; visualisation tools

Participants were also asked to discuss ‘extreme/high impact events’ and ‘behaviour’.

- **Extreme/high-impact**: sensitivity of extreme event projections to changes in model structure; verification of forecast probabilities for extreme events; can “tipping points” be recognised in advance?
- **Behaviour:** agent-based modelling; integration of environmental indicators to feed into societal benefits; use of behavioural models from economics.

4. Connections between the challenges and mathematical sciences identified

In the final facilitated discussion participants were asked to consider any connections between the environmental challenges and mathematical sciences identified during the workshop, this included groupings that could come together, similarities and differences. In addition, they were asked which stakeholders should be involved in further discussion and activities.

Themes running through the discussions included:

- **Data**: quality, availability, missing data, big and small data
- **Use of models**: experimental design, how models are interpreted, validation/verification
- **Non-autonomous nonlinear mathematics:** multi-level and multi-scale models, integration of models, feedback between models, acknowledgement of model restrictions
- **Uncertainty**: managing the decision process, across scales and time, communication of uncertainty
- **Unexploited mathematics:** mathematics of relevance to real world applications e.g. behavioural science

Notes of this discussion are available in appendix 5.

Stakeholders which participants felt should be engaged with included other research councils, government departments (such as DECC, DEFRA, DfT, DfID, CCC, ONS), the insurance industry, economists, Met Office, CBI, social scientists. Methods suggested for doing this included focussed workshops and public engagement.

In order to inform EPSRC’s thinking participants were also asked whether there was potential for a funding activity at the mathematical sciences/LWEC interface and what the best use of any funding might be to encourage development of the area.

General observations

It was noted that additional input should be sought from experts in economics, business forecasters, social sciences, bio-diversity, biochemistry/biogeology, scientific software engineers, high performance computing/scientific computing and numerical analysts.
The following points were noted on the ‘thoughts’ board:

- Who is best qualified to formulate the challenges resulting from environmental change?
- Good data is crucial
- Stay aware of limitations of science in this area, don't oversell
- Don't rule out a sandpit as a funding activity in this area
- There is cross-council interest in this area e.g. dynamical modelling (EPSRC) of environmental challenges (NERC) requires input from economists (ESRC) to inform adaptation and mitigation strategies.

The energy in the room to contribute and share ideas was extremely high. This can be seen in the appendix and covers many varying and diverse agendas. However there was a lack of overall convergence on specific ways in which the two communities, mathematical and environmental science, could align opportunities and challenges.

EPSRC expects that the developed activities will be engaged with broadly and will lead to further development of the identified topics.

Next Steps

The workshop was the first step in allowing a diverse community to feed in thoughts on the topic of forecasting and elucidating how mathematical science can impact on environmental change research. Outputs have identified that there is much to explore and there is much will to explore this cross-disciplinary research space. However after the two day workshop it was apparent that there was a lack of clarity on the specific research themes and challenges that needed tackling.

To continue the conversation and identify focussed research opportunities EPSRC plans to launch a call for Networks on themes made explicit at the workshop.

Contacts

For further information about the workshop, please contact:

Vivienne Blackstone
Healthcare Technologies Portfolio Manager
Vivienne.Blackstone@epsrc.ac.uk

Iain Larmour
LWEC Portfolio Manager
Iain.Larmour@epsrc.ac.uk

Hannah Maytum
Mathematical Sciences Portfolio Manager
Hannah.Maytum@epsrc.ac.uk

Christopher White
LWEC Senior Portfolio Manager
Christopher.White@epsrc.ac.uk

Appendix 1

Workshop participants

Name		Organisation
Peter	Ashwin	University of Exeter
Robert	Beardmore	University of Exeter
Margaret	Bell	Newcastle University
Vivenne	Blackstone	EPSRC
Peter	Burlinson	BBSRC
Frances	Collingborn	NERC
Matthew	Collins	University of Exeter
David	Elston	BioSS
Julian	Faraway	University of Bath
Christopher	Ferro	University of Exeter
Michael	Gastner	University of Bristol
Sotos	Generalis	Aston University
Jacqueline	Glass	Loughborough University
Philippa	Hemmings	EPSRC
Peter	Henrys	Centre for Ecology & Hydrology
John	Hillier	Loughborough University
Richard	Hindmarsh	British Antarctic Survey
Rebecca	Hoyle	University of Surrey
Georges	Kesserwani	The University of Sheffield
Markus	Kirkilonis	University of Warwick
Nikolaos	Kourentzes	Lancaster University
Robert	Lamb	JBA Trust
Iain	Larmour	EPSRC
Amos	Lawless	University of Reading
Joseph	Leedale	University of Liverpool
Mike	Lonergan	University of St Andrews
Hannah	Maytum	EPSRC
Markus	Mueller	University of Exeter
Tim	Palmer	University of Oxford
Sarah	Parker	University of Oxford
Sandhya	Patidar	Heriot-Watt University
Ian	Pattison	Loughborough University
Gareth	Pender	Heriot-Watt University
Ramakrishnan	Ramanathan	University of Bedfordshire
Monica	Rivas-Casado	Cranfield University
Tom	Roberts	ESRC
Tim	Ryley	Loughborough University
David	Schultz	University of Manchester
Marian	Scott	University of Glasgow

Roger	Singleton Escofet	EPSRC
David	Stainforth	London School of Economics
Erica	Thompson	London School of Economics
Jacques	Vanneste	University of Edinburgh
Jochen	Voss	University of Leeds
Keith	Weatherhead	Cranfield University
Chris	White	EPSRC
Ruth	Wood	University of Manchester

Appendix 2

What forecasting means to you

Table 1A

Flood depth
£
Happiness
Cost-benefit (i.e. metrics)

- Direct
- Indirect (e.g. links/networks)

Single (e.g. Poisson events)
Complete hazard (e.g. interacting, time-lagged)

Management
Decision-making
Intervention
↓
Linking models
↓
High dimensionality-challenges
Deterministic/stochastic
↓
Integrating
Forecasting

→ Not one then the other

Tools, e.g. management

Model independence

Multi-dimensionality – 1D ↔ 2D ↔ 3D – Downscaling for flood models
Multi-mode ensembles? to increase the confidence for decision-making

Agent-based modelling
Event set modelling

Initial and perturbed physics scenarios
'event-set' (all possible scenarios)
↓
Deep uncertainty
→ Aleatoric and epistemic
Uncertainty ('end-to-end') Quantific?
Probability-distribution
Ensemble methods
Sub-sampling (?!?)
Tail-end extreme
'Event-set' (all possible scenarios)

From decisions to experimental design – skills evaluation

Designing models and <u>model experiments</u> to address specific policy decisions

Extrapolation is dangerous

Verification – deterministic and probabilistic

Calibration

Model initialisation - observations

Table 2

Accuracy
Prediction
Identify process
Determine pathway
Analyse
Symmetry(ies)
Interaction
Boundary conditions
Boundary conditions
Bifurcation analysis

TIMESCALES

- 1 day (e.g. flood event)
- 1 'season' – crops
- 1 year (insurance)
- 20 years (housing stock planning – location/design)
- more than 200 years

- Impact for decision making
    - Forecast uncertainty
    - Quantifying uncertainty
    - Short - long-term prediction
    - Choice of relevant variables and models
    - Forecasting methods
    - Predictive monitoring
    - Event alerts
    - Trend changes and break points
    - Endogenous and exogenous systems of variables/information
    - Forecasting with limited data
    - Policy and forecasts
    - Point forecasts vs. prediction intervals
    - Translation of forecasts to stakeholders
    - Judgement in forecasts
    - Scenario building

Strategic
Tactical
Purposes ↔ questions to be answered
Climate and weather
- direct impact
- indirect impact

Impact on:
- natural environment

- human environment
- economy
- food production
- transport networks
- automation flow
- energy
- zoonotic discussion
- equality – human justice
- policy maker – short term/long term
- self votes/somebody else's problem

Stochastic probability

- methods modelling
- predictions – uncertainty – input data → quality / quantity / support / resolution

Climate change
Error propagation

- spatial context
- visualisation tools
- geographical information systems

Multidisciplinarity

- environment
- engineering
- mathematics
- statistics
- state of the art technology
- social science and social impact – communication tools – fit for purpose

Table 3

Model

- data
- scenarios
- state at time
- distribution
- ensemble of i.c
- initial conditions
- t = p

Forecasting for:

- weather)
- climate) dependence on conditions
- populations
- anything
- economics
- pestilence
- earthquakes

Rational for forecast can determine the model structure or inputs e.g. protective or proactive decision making

Data inputs to forecast models

- actual observations
- relationships between factors
- coarse data – filling in the gaps
- appreciation of unknown and uncertainties
- communication and interpretation of model and data input uncertainties
- scenarios vs. probabilistic outputs

Personalised forecasting of response to a drug

Prediction vs. forecast vs. projection

- difference between them
- how far into the future?
- drivers of change
- scenarios – stochastic or deterministic
- technology advances – game changers
- forecast/prediction verification
- forecasts as input to other 'models' – decisions planning
- interaction

Table 4

Communicate (in confidence) uncertainty (error bars or one value) which is most valuable to stakeholders

Improving forecast as event nears, eg max flood height (responsive) using past data

- adaptable models that run quickly
- continuously updated

Sources of information

- social networking/crowd sourcing
- sensors
- past data

What makes a good forecast? For who?

Improving forecasting

- forecast the process
- assess forecast performance
- observe the process
- create/issue forecast product
- assess utility
- improve forecasting system

All steps are crucial to producing effective forecasts!

Unpredictable" events?

Past data insufficient for adequate prediction. How can this be supplemented? (experiments and judgement)

Monte Carlo techniques can be used to capture low probability events

Distinguish between physically-based models (weather/climate/hydro) where extreme future states can be more easily forecast than economic/social/human forecasts where new equations may be needed

Merits of designing models that can predict low-probability events?

Validating long-term forecast

Limitations of using past data as a means of validation for a <u>predictive</u> model

Error. Sensitivity of models. Actual variation uncertainty

Understanding the changes in parameters over time in the past helps inform models to estimate probability of "events" in the future

However
The extreme event may be due to something/influence not expected/experienced before
Therefore
Delivery risk – adaptable models over investment in solution getting it wrong
System based on historic/observed data used to touch (?) model for set of parameters.
Then combination of these parameters and to generate range of possible outcomes that could be manifested in form of forecasting distribution (forecast has to be probabilistic)

Table 5

'Living with Environmental Change' – acting on certain information. What is good metric for change?

'Acting' on uncertain information

What is good metric for "change"?

Stakeholder interpretation:

- telling us the future, very precisely
- point predictions

vs.

Scientist/forecaster view

- not necessarily point predictions
- what <u>could</u> happen, not always what <u>will</u> happen (relates to stability)
- uncertainty
  - probabilities
  - structural error
  - what is not in the model?

What is the objective of the model? (e.g. define 'cost'/impact)
Trade-offs
Needs to be defined in advance (and how accurate does it <u>need</u> to be)?
And what if it "needs" to be more accurate than it <u>can</u> be?
Conditional vs. unconditional forecasts
- what are they? predictions/projections
- using right terminology

Way presented to audience
↓
Limit of modelling process
Communication issues
Human response to uncertainty
Aggregation of ("social") models

End-to-end drivers
Joining up all the dimensions
May be very inter-dependent, eg flood risk
- PDES
- Agent-based models
- Policy
- Human behaviour
- Etc.

→ Mathematical modelling

Aggregating influences
Subjective (instruction manual for how to be subjective)

Understanding uncertainty – responses to uncertainty
Multidimensional problems – many alternative views – positioning?
Beyond scenario building – are you providing point forecasts?

Forecasts as thinking tools, not just numbers
→
Acknowledging and embracing subjectivity in scientific modelling
Pitfalls of claiming or implying "objectivity" especially in extrapolation

Uncertainty about things that might change whole basis of model ("Big Surprise")
Challenging models with data – has to be possible (not always)
Big Data ??
Small Data?!
Complex Data

Table 6

No matter what forecast prediction you provide, how do you get people to believe (or not believe) it?
- very important point can bring in end-users to the design process but this is difficult to get engagement through a seemingly slow/long research timescale

Validation
Communication tools
Modelling the 'people' component
Understanding uncertainty; and, how policy makers incorporate it into decisions/different parts of the system
What are the assumptions in a forecasting model? → Nice → understanding models basis
Forecast/model business impacts/consumption patterns/Government regulations
Is the maths adequate and is it the concepts behind forecasting that are inadequate?

It can be a problem if "it all looks right" when it isn't complete
"Good enough" what is enough?
Timescale differences – Government vs. individuals and companies
Would agent-based help with variability/uncertainty (same way event-set models could – by simplifying)
Agent based modelling
Uncertainty not just from climate change (input data) or imposed boundary conditions
Business decision timescales shorter than legislative timescales
Can we use flood risk models to forecast consumption patterns of people?
Supply chain impacts?

Appendix 3

Key environmental challenges

Uncertainty and Variability

- Adapting to (greater?) variability
- Modelling and quantifying uncertainty and variability
- "Uncertainty"
  - Recognition different types in models
  - How it impacts on decision-making at government/regional/business/individual level
  - The translation of economic uncertainty to calculate TRUE COST
  - Impact of technological change on system e.g. rebound effect? And long-term behavioural change?
- Data
  - Confronting models with data.
  - Using models and scientific understanding to direct observing campaigns
  - What data do we need?
  - Continued time series
  - Smart technologies/privacy
  - Assessing the value of different data.
- Numerical Aspects
  - Definition of terms e.g. what does forecast mean.
  - Time scales of different processes involved in climate forecasting
  - Computational cost of climate simulations
  - Solution techniques of Navier-stokes equations
  - Decide on models to be employed for solving N-S equations
  - Uncertainty related to numerical solution schemes
  - Quantifying uncertainty – representing / communicating, visualising
  - Dependence and high dimensionality
  - Cryosphere

Integrating Models/Multiscale Models

- Multi-scale systems. Horses for courses. Measurements as well as models.
- Linked models – modular/flexible
- Improve the modelling of CO2 emissions in climate change modelling
- Climate change -> (modellers, academic scientists, share outputs to make difference) -> Communication -> (engineers and scientists, Behaviour change) -> Causes – Anthropogenic -> (Improve model integration) -> Back to Climate change (it's circular)
- Interactions between different challenges – boundaries, link between many models.
- Integrating different themes within environmental change.
- Multi-objective models/projects – Tracing assumptions, uncertainty, probabilities between disciplines – Metrics!
- Environment Commonalities
  - Land use (spatial) – agricultural/urban and infrastructural.
  - Social challenges
  - Sea level change – infrastructure (spatial), adaptation, mitigation (uncertainty)
  - Water resources (spatial) – too much in some places, too little in others. Floods/drought

    - Environmental resources (metals, soils, biodiversity) – limited in quantity (spatial/integrating across scales)
    - Ecosystem services
    - Drivers of change
- Spatial/Integration over scales/design.
    - Dealing with time varying spatial boundaries/domain
    - Nesting of scales
    - Hierarchical modelling
    - New technology – volume of data, identification of events, design of "observational" frame.

Decision Support and Communication

- Communication around uncertainty in forecasting
- Definitions of variation, sensitivity, uncertainty, forecasting etc. mean different things to different people – standardise.
- Communication – scientists, public, maths, policy makers, modellers.
- Decision Making
    - Adjust science to policy context
    - Spatio-temporal scale of forecast
    - Communication tools and methods required (for forecast and uncertainty) specific to target audience.
    - Tools that translate probability and uncertainty into economic/policy terms.
- Making outputs relevant for decision making
    - Time and length scales
    - Provision of uncertainties.

Extreme/High-impact Events

- Black swan events/tipping points/ice age
- Unusual events/changes
    - Effect of "tipping points" (collapse of ice sheets,…)
    - Change points/interventions and the effects on detection of change/unexpected…
    - What interventions drive change? How do we model them?
    - What we don't know
    - Understanding the time it takes for the atmosphere to reach equilibrium with new CO_2 level.

Behaviour

Frameworks for understanding and quantifying epistemic uncertainties

- Behavioural "Modelling" (?)
    - Policy
    - Understanding behavioural trends
    - Influencing behaviour
- Modelling behavioural responses (to uncertainty?) (Feedback?)
- Societal Impacts
    - Engineering
    - Human Behaviour/society
    - Biodiversity – agri/food
    - Health
    - Designing society – individuals -> global collective, agent based modelling

Population Change

- Migration
- Access to resources including food.

Resource Management

- Scarcity
- Economics
- Food/food cycle/nitrogen cycle
- Water
- Renewable/non-renewable resources – Energy

Only One Real World

- Linking models to policy, decision strategies and investments
- N = 1
- Handling the high percentages of the “variable(s)” of interest – not about mean/modes.

Sources of Pollution

- Reduce CO2
- Water security
- Range of pollutants e.g. NOx
- Noise to local – people
- Transport emissions – policy/behaviour
- Air quality – multivariate
    - Multiple pollutants
    - Risk based approach for single pollutant at a time.

Appendix 4

Mathematical Sciences contribution to challenges in environmental change
Uncertainty and Variability

- Ensembles
  - Of model runs
  - Of model types
  - Analysis of ensemble outputs
- Non-autonomous systems mathematics
  - Attractor properties changing parameters? E.g. variability changes.
- How much uncertainty is due to lack of data?
- No amount of fancy statistics can make up for lack of data
- Are we making best use of the data we have?
- Bayesian filtering
- Does the wider community know what data is available?
- Propagation of uncertainty along model chain.
- Intrinsically linked to scale.
- Can you distinguish between inter-annual variability and trends? Will be different for different predictands.
- Types of uncertainty – parameters, model structure and nature.
- Extremes versus average (exceedance)
- Reducing number of simulations – intelligent experiment design (parameter space)
- What are the experiments in above?
- Cross reference to chemometrics – methods for identifying key experiments to characterise a broader range of possible outcomes.
- Perturbation analysis.
- How is uncertainty in data sets and their manipulation amplified/cascaded through complex/integrated models.
- Coping with irreducible uncertainties.
- Conditionality of uncertainty quantification.
- Long range dependence and bunched black swans.
- Initial values -> Model (lack of fit/model error) -> Outputs (probability distribution of modelled quantities, probability drift of observations)
- In above – initial values – perturbations due to uncertainty/incompleteness/indirect relationships with modelled quantities. Model – Distribution of (estimated) parameters and ongoing observations. Outputs – How represent pdf for high dimensional systems? What is most useful way to visualise/communicate?
- Risks of subsampling
- Bootstrapping?
- Is the model right? No! But it might be useful (after Box)
- Integrate "internet of things" into modelling.
- Using big climate models vs testing ideas using simple models -> impact on e.g. IPCC
- Use of "expert opinion" in models
- Correlations – spatial/temporal
- Limitations of methods
- Validation
- Fuzziness – definition based on measured data?
- Ensemble methods
- "Equation-free" modelling – as technical term relating to using simulations to do dynamical systems – type of things like producing bifurcation diagrams.
- Different methods applied to obtain solution

- Model/Modeller biases – do we consider the correct model form?
- Probability, stochastics
- Time-series analysis – spatial analysis
- Modelling – deterministic approach, physical vs statistical, stochastic
- Numerical methods – Monte Carlo, parameterization
- Control theory (geoengineering)
- Modelling pdfs – distinguishing model-pdfs from real world pdfs
- Extreme Events
- Epistemic uncertainty (model structure)
- Subjectivity; making assumptions clear.
- Linking of microscale outputs to feed into higher scale models
- Integration over scales

Integrating Models/Multiscale Models

- Enabling two-way feedbacks between each component of an integrated model (e.g. model based geostatistics.) (Yes)
- Monitoring and data collection – models should inform on where data need to be collected next to generate better/more accurate model outputs.
- Observation data may also be at multiple scales.
- How low do you go? Chemistry/microbe?
- Uncertainty in input data? -> model resolution
- What "data support" is appropriate for the model?
- Model parameters
- "Uncertainty propagation" when combining models, linked to scale -> develop useable tools.
- Upscaling?
- Downscaling?
- Multiscale techniques (homogenisation, averaging…)
- Which scale is appropriate? Downstream impacts that are not obvious?
- Spatial Scales? -> data fusion, statistical +….
- Temporal scales? -> data assimilation
- Temporal scales? -> computational/numerical techniques appropriate to multi-scale models? -> big data issues? -> appropriate statistical techniques?
- Choice experiments? Useful in communicating models to decision-makers?
- Validation of models and issues of "Trust" – agree, look at signalling theory.
- Compatibility of models, data/parameters mean same thing, processes etc. not repeated.
- Multiscale partial differential equations.
- PCA/factor analysis (exploratory technique only (Danger, Danger)
- Traceability of model-variables between alternative model structures
    - Identifying hierarchies of models and outputs.
    - Identifying correlations/patterns/relationships
    - Understanding what model variables are.
- Multiscale analysis
- 1950 Theodorsen N-S
- Bottom-up computationally intensive <-> link to top-down?
- Fuzzy matching/stochasticity?
- Phenomenological Metamodels <- detailed models feed in.
- Designing models specifically so that they can fit together (plus end user relevance for al sub points below)
    - Data formats
    - Time and spatial variables
    - Choosing the right model variables

    - Propagating uncertainty
    - Diversity of models
- Reconciling accuracy and complexity of the models
- Reconciling model output differences
- Aggregating variables in complex systems – e.g. healthcare many different types of information (?Manchester?)
- Continuation of initial conditions (is a figure for transport but will try and capture it below)
    - Demographics as base -> traffic next layer -> Emissions, CSEQ (carbon sequestration) and Energy Build (the net output of this layer gives a NET CO2 emission level which is an input to the climate change modellers as a separate node) -> next layer relates to the dispersion of the emissions -> Dose/response layer is next -> top layer is health (NDBF, RAH [not sure if these are the right abbreviations)

Decision Support and Communication

- Understanding connotations of language. Thesaurus: uncertainty ≈ doubt, confusion, mistrust,…. Use e.g. "range of projections" instead?
- Well posed options/questions.
- Representations of uncertainty clearly include the unknown unknowns <- and assumptions/caveats <- new maths needed for nonautonomous nonlinear systems.
- Choosing meaningful variables.
- Communicating the limits of available model info.
- Helping government/business/supply chains to make informed decisions.
- Real options techniques
- Can economic theory be used to inform decision making theory?
    - <- Not simplified point predictions use full probabilities but communicate easy to understand decisions.
    - <- about "values" as well. See ecosystem services, values nature.
- Bring uncertainty of climate models into economic models/tools into optimisation models into decision support tools <- e.g. insurance "catastrophe models" currently do this (OK very simplified)
- Mathematical sciences in decision support and communication.
- New applications of maths to world problems – not necessarily new "maths" needed to engage decision makers, but applications to guide development of techniques.
- User/public engagement in this area – what works? <- Inclusion of decision makers (stakeholders) in "doing" question/research
    - Ask right questions
    - Collect/model right things
    - Right data
- Two way communication needed between the researcher and stakeholder ("end to end to end" model) <-> Defining the key outputs to be communicated to meet user needs is essential. <- what if user "needs" (wants) something that models can't provide? E.g. postcode predictions of weather in 2080.
- Fuzzy Set Theory
- Frameworks for structuring knowledge
- We (society) needs/why? For – modelling environmental change, explaining, managing.
    - Framework of physical/environment system (not just climate) forecasting

    - Design of framework for observation/data collection -> spatial/temporal/multivariate and integrated (so interactions), People matter – so behaviours
    - Statistical/physical models
        - Hierarchy of scales
        - Drivers -> a focus on uncertainty in all the variables so joint distributions rather than marginal.
    - Scope to investigate 95% or 5th% rather than mean/mode.
    - Improved capability in merging/linking
- Would RC support 1-year collaborative book-writing fellowships in environmental change/maths?
- Translation
    - Management of risk + investment planning.
    - Transparency
    - Solvency –EU directive requires transparency in catastrophe modelling (Big and thick!!)
    - Policy
- Cost and benefit definition (for whom?) + metrics.
    - Non utilitarian decision support tools.
- Elicitation methods with probability
- Optimization methods and dynamics
- Deterministic approach -> multi-criterion and multi-objective.
- Stochastic risk.
- Portfolio management
- Stochastic event sets
- Clustering
- Communication format -> influences, responses,
    - Model? Agent Based Modelling (ABM)
    - Opinion dynamics?
    - Network dynamics>
- Visualisation tools
- Visualisation/animation
- Algorithm development software?

Extreme/High-impact

- How to model what you can't know your model needs?
- Difficulty in attributing extreme events to drivers
- What to expect, with and without changes in forcing. Bunched black swans.
- Do thresholds (exceedance) change over time?
- Sensitivity of extreme projections to changes in model structure.
- High impact for some might be low impact for others.
- How do we measure "high", "extreme".
- Lack of data
- Can "tipping points" be recognised in advance?
- Clustering of rare events in space/time – preconditioning atmosphere in weather regimes.
- Extrapolating the frequency and intensity of rare events over a long time series, given a short observational record.
    - Engineering solutions (what does this mean? Is it a piece of equipment?) required to collect data during extreme events and efficiency of data collection.
- Attribution of climate change for high-impact events -> mathematical approaches to determine attribution.

- Forecasting!
- Communicating deep uncertainty (non-probabilistic)
- Risk awareness -> how to measure at society level.
- Extreme dynamics are required in order to identify weaknesses in model structure.
- Resilient policy making.
- Frequency/severity (quantifying).
- Probability of events too rare to be seen in observational record.
- Adaptability
- Use scenarios to engage/provoke imagination and political will
- Even when (if) you know the probability, how should we respond to very extreme but very rare events?
- Numerical methods (+ [geo]engineering solutions) for sampling rare events.
- Verification of forecast probabilities for extreme events
- Importance of sampling – dangerous – can easily miss the extremes.
- High impact, low probability events e.g. Amazon dieback, AMOC shutdown.
- Need to develop engineering solutions to sample/collect data during extreme events.
- We cannot model ice adequately!
- Cannot defend against all possibilities – which ones to ignore?
- Understanding Milankovitch forcing of ice ages.
- Extremes of forces possible on buildings/infrastructure.
- Selection bias in modelling the extremes that have been observed but not focussing on ones that haven't (yet).

Behaviour

- Test response to (policy) interventions.
- Is <u>individual</u> human behaviour predictable at all?
- Behaviour is not the y-variable to be predicted, it is the independent x-variable to be chosen! (or manipulated by policies = predict outcome)
- Is using average behaviour adequate? (No – use ensembles….)
- How to change people behaviour to facilitate diffusion of environmental friendly technologies?
- Policy?
- Explore consequences of particular behavioural responses e.g. <u>agent-based</u> modelling/Monte Carlo on networks.
- Integration of environmental indicators to feed into societal benefits.
- Can cluster analysis enable us to bridge the gap between modelling the individuals and the whole nation as one.
- Agent-based modelling and social science.
- What is the mathematical equivalent of a rich picture? (soft systems methodology – can they complement each other?)
- Can fuzzy logic/agent based modelling help us understand how behaviour/practices may change demand for resources over time?
- Role of game theoretic approaches to shed light on mitigation activities.
- Role of social – i.e. link to social networks of people and psychological – i.e. how people's attitudes affect behaviours. As well as economic approaches/models to understand behaviour.
- Role of belief systems and religion in behaviour.
- Religion is a "no-go" area at present. If x% of Americans really believe the world ends in 20..?, how will that affect how they will react? And how do we model that? -> Google "The Arch Druid Report"
- Agent-based modelling.

- Economics has lots of behaviour/models. Are they any good?
- Is ther any value in looking at space syntax (UCL) model for transferability to behaviour?
- Are models (each) designed to inform or to motivate?
- Essential to translate scientific outputs to stakeholders who can incentivise/deliver policy to change behaviours.
- Develop model framework to be “fit for purpose” to reflect the impact of behavioural change and “close the loop”

Appendix 5

Connections between the environmental challenges and mathematical sciences identified

- Data
    - Quality
    - Availability
    - Missing Data
    - Directed Data Gathering
    - Big Data
- How we use models
    - Interpret
    - Experimental Design
- Non-autonomous non-linear maths
    - Multi-level
    - Multi-scale
    - Cross-disciplinary linkage
    - Integration of models
        - Feedback between models
        - 2-way coupling – but we shouldn't try to model everything.
- Keeping track of model assumptions and uncertainty – being aware of "black box" (keep it small)
- Acknowledge restrictions of model downstream.
- Knowing where in multilevel model feedback loops are.
- Via sectors
- Uncertainties – and managing decision process
- Managing uncertainties across scales and time
- Need to be thinking of the bigger/wider picture
- Implication of changing climate on other things
- Missing feedback loops
- Mathematics of behavioural science (average behaviours change)
- Simplifying complexity
    - Multiscale
    - Sampling techniques for models and extreme events "Important Sampling"
- Mathematical training for scientists and others
- "Interactional expertise"
- Also consider 2 way training with users
- Speed dating with mathematical techniques
    - Link data problems with methods
- **Intra**disciplinary possibilities
    - Encourage deterministic and probabilistic approaches
- Communication in models with impacts of decisions.
- Astrophysics/cosmology modelling applied to Earth forecasting
- Data assimilation/integration into models
- Big data/software (too much data?)
- Performance monitoring
- Uncertainty classifications/communication
- Reanalysis models
- Small data – verification problems
- **Mathematics that (is novel to agencies/unexploited) have relevance to a wide (≥1) number of applications (novel applications). Novel to who?**
- Themes, pick funding depending on theme.
- Develop programme

- 3 avenues prioritising
    - Behaviour (interdisciplinary)
    - Encourage in responsive mode
    - High risk call
- More representation from ESRC community
- Operational decision making help.
- Maths connected to something else (real world)
- Network between areas already close.
- Tools still not known what is best, other disciplines needed to feed in.
- Understanding between disciplines needed.
- Difficulty in identifying challenges
- Ecosystems
- Scientific computational models/updates
- Data validation/verification/How will this be done?
- Environment broader than climate change – this might not have come across.