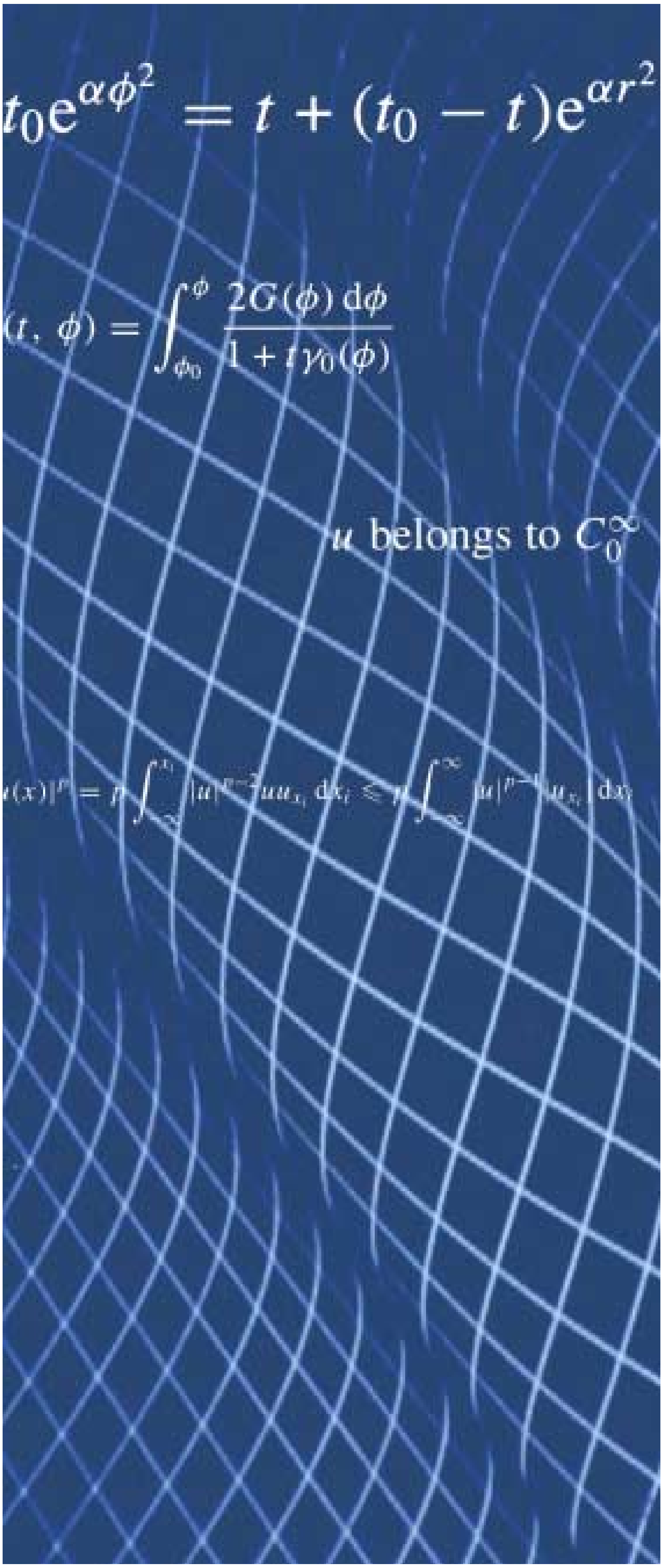
An International Review of UK Research in Mathematics



A Review undertaken on behalf of the Engineering and Physical Sciences Research Council (EPSRC) and the Council for the Mathematical Sciences (CMS)

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8 March 2004

Foreword

This report of the International Review of Mathematics Research in the UK was prepared by a Panel of world-leading mathematicians and statisticians. The review is the sixth in a series commissioned by the Engineering and Physical Sciences Research Council. Each is conducted in partnership with the relevant learned and professional bodies, in this case the Council for the Mathematical Sciences (comprising the Institute of Mathematics and its Applications, the London Mathematical Society and the Royal Statistical Society).

We welcome this report and its analysis of the strengths and potential of mathematics and statistics research in the UK. In addition to making detailed comments on the current state of research it addresses the issue of how the UK can maintain and improve the health of its research base into the future. It points to areas in which the UK's world-leading role must be preserved, and other areas, of particular importance, where the UK community and funding organisations should work in concert to take actions and develop special initiatives to strengthen the UK's position.

We commend the report to the mathematics and statistics community. We should like to see it engender a broad, inclusive debate on the issues it raises and the ways all of us – the funding bodies, the mathematical societies and the research community – can address the issues and work together to preserve the health and vitality of our subject.

We wish especially to express our thanks and admiration for the work of Jean-Pierre Bourguignon and his Panel in undertaking the review and producing this report. We were extremely fortunate to have such a distinguished and expert group, with its broad, international perspective on mathematics and statistics. We are also most grateful to Martin Taylor who led the Steering Group for the Review, to Stephen Huggett, the Scientific Secretary, who organised and supported the Review, and to Peter Cooper, who as CMS observer contributed greatly to its smooth running.

It is now the task of us all – the mathematical community, the funding bodies and the mathematical societies – to build on this Report to ensure that the UK maintains its place at the forefront of mathematics and statistics research.

John O'Reilly
Chief Executive, Engineering &
Physical Sciences Research Council

Frances Kirwan
President,
London Mathematical Society

Tim Pedley
President, Institute of Mathematics
and its Applications

Andy Grieve
President,
Royal Statistical Society

23 March 2004

The International Panel (left to right): Jean-Pierre Bourguignon, Robbert Dijkgraaf, Hans Othmer, Stephen Davis, Stephen Huggett, Margaret Wright, Ron Graham, John Guckenheimer, Niels Keiding, Michel Broué, Susan Murphy, Don Dawson, Peter Hall. Inset: Peter Sarnak.

General Context and Scope of the Review

"How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?"
Albert EINSTEIN

The contemporary position of mathematics: new challenges and opportunities

This age is one of remarkable accomplishment in the mathematical sciences.

In pure mathematics, the most visible signs of progress are proofs of major, often longstanding open questions, notably the widely-publicized proof in the mid-1990s of Fermat's Last Theorem. During the past few years, there are indications that the Poincaré Conjecture in topology has been settled. In both of these cases (and in many others where breakthroughs have occurred), the final step—completion of the proof—critically depends on theories and research from several areas within mathematics, suggesting an increasing unity within pure mathematics.

In applied mathematics, progress is recognized in a variety of forms: deepened understanding of physical phenomena such as turbulence, creation of new mathematical constructs that capture previously unrepresentable behaviour, and development of new algorithms that allow problems to be solved numerically for the first time (or much faster than before). An example from everyday life is that three-day weather prediction is more accurate today than one-day forecasts were 15 years ago because of significant improvements in all these aspects of applied mathematics.

Statistics, sometimes described as the first and most successful information science, involves synthesizing and drawing inferences from data and experiments. Important developments in statistics have often been inspired by real-world applications, especially in medicine, the social sciences, agriculture and economics. Statistical thinking, and its associated theory, have changed our lives. For example, Bayesian ideas are used in a great many modern technologies, from camera light-metering systems to methods for assessing credit-worthiness. The efficacy of many achievements in medicine is assured only through modern advances in statistical analysis.

In the mathematical sciences viewed as a whole, new fields continue to emerge that blend two or more classical fields, with the overall effect that formerly firm traditional boundaries are crumbling.

Mathematics—by which we mean pure and applied mathematics as well as statistics—is a discipline in itself, but it also provides a language and a vast array of tools for describing, analyzing, and solving problems in other fields. It goes almost without saying that mathematics has influenced and continues to influence new and far-flung areas of engineering and science, ranging from systems biology to building bridges, magnetic resonance imaging, and financial option pricing, confirming a statement by the "Father of Flight", engineer Theodore VON KÁRMÁN, *"There is nothing more practical than a good theory"*.

Mathematical and statistical models, combined with numerical methods, allow scientists and engineers to identify and extract dominant mechanisms of behaviour from large complex systems, and to begin to understand how sociological, economic, natural, and industrial systems operate. Moreover, this is a two-way street: the need for new techniques in applications leads to the development of new research areas in mathematics. Thus research in mathematics lies at the heart of the way science shapes contemporary society. General W. E. ODOM, former director of the National Security Agency (USA), observed in a US National Science Foundation report about mathematics research that *"Mathematicians... make frequent breakthroughs, and the consequences for all other branches of science can be enormous, not to mention the technological and economic gains that follow."*

Some recent substantial contributions of mathematical research to society

The following list is meant to highlight only a few of the numerous ways in which mathematics research has an effect on daily life.

As early as the Second World War, British mathematicians initiated a methodology for *cryptography* and security. The protection of information is now everybody's concern, in contexts ranging from credit cards to national security. Cryptography's foundation in what was once the purest of pure mathematics led the British mathematician Andrew WILES, famed for his proof of Fermat's Last Theorem, to comment that *"one change in number theory over the past 20 years is that it has become an applied subject."*

Game theory, with roots in mathematics, statistics, and economics, is routinely applied today to understanding and predicting human behaviour. Game theory recently played a pivotal role in the design of auctions for third-generation mobile telephone licences in the United Kingdom (UK).

Medical imaging depends on solving *inverse problems, on approximate reconstruction strategies*, and on *signal denoising*. Thomas F. BUDINGER, a medical doctor and bioengineer, stated in 2003 that current endeavours to improve resolution and sampling speed, decrease background, and achieve reliable quantification in medical imaging *"rely on innovations in mathematics applied to the inverse problem."*

Numerical methods for *partial differential equations* have advanced in both accuracy and speed based on innovations in theory, matrix computations, and adaptive algorithms. A notable application is the design of extraordinarily efficient airfoils.

The mathematical theory of *wavelets* enables very efficient signal compression, with applications ranging from the video entertainment industry to fingerprint analysis to many areas of science and technology. Wavelet methods are fundamental to the new standard for image transmission, JPEG4, ensuring its widespread circulation in the professional world.

The application of *stochastic analysis to the financial industry* has resulted in the creation of multi-trillion dollar security markets that have facilitated the international flow of capital and contributed to commerce and productivity. The development of reliable pricing methods for derivative securities that are used for risk management would have been impossible without the application of probabilistic models and stochastic analysis that have been developed over the past thirty years. This is only one example of the many contributions made by probability theory to predicting outcomes in stochastic environments.
The impact of modern probability and statistics is everywhere: in the manifest benefits to society from agricultural field experiments, in the use of stochastic models to produce better communications networks, and in the probabilistic analysis of the risk of unlikely but catastrophic events...

Data mining, the determination of patterns and anomalies in data, combines statistics with pure and applied mathematics as well as computer science. Analyzing and learning from data will be of increasing importance because the sheer volume of data associated with numerous applications—astronomy, biology, medicine, geoscience, and atmospheric sciences—is growing at astonishing rates. Predictions abound that rapid, perhaps real-time, analysis will soon be needed for petabyte (10^{15}) data sets.

Finally, there is no field in the foreseeable future where mathematics will have a greater impact than in the life sciences. Four of the eight recommendations of the US report BIO2010, commissioned by the National Research Council, call for closer collaboration on the basis that *“the connections between the biological sciences, and the physical sciences, mathematics and computer science are rapidly becoming deeper and more extensive.”*

Mathematical research: a people-intensive activity

Mathematics is without any doubt a part of science, but its mode of operation has intrinsic special features. Because generality and abstraction are key ingredients in mathematics, deep mathematical ideas display extreme longevity in their influence, sometimes returning to prominence after many years in abeyance. As a result, there is an unusually strong connection between the present and the past in mathematics compared with disciplines that are more dependent on advances in technology or equipment. Another feature of mathematical research is that profound contributions are very often made by individuals or relatively small groups of people, so that individualism remains a bedrock of discovery in mathematics. This is not a universal truth, however: mathematical scientists who work directly with applications sometimes belong to teams of medium and even large size.

Research in pure mathematics and theoretical areas of applied mathematics and statistics tends to be built around core groups of people, as distinct from facilities or laboratories. However, it is not at all the case that mathematicians can thrive without equipment, since software and computation are the usual means by which mathematical ideas are transformed from research to tools, for mathematicians as well as for users from other fields. In addition, powerful computers are an essential part of a wide spectrum of mathematical and statistical research—for example, in genomics, applied cryptography, checking the Riemann hypothesis, and simulation of complex physical and biological systems.

Another feature of mathematics, as a discipline, is that there is an almost immediate relationship between research and teaching. The artefacts of mathematics are results, insights, algorithms, techniques, and software. These forms of knowledge permit very rapid introduction of the latest research in the mathematical sciences to applied fields of science and technology through the training of graduate students. However, dissemination can occur at a fast pace only if the human capital is at hand. Thus the ability to attract enough talented young people is a central issue not only to the growth of mathematics but also to the health of an increasingly large part of the high-tech world.

The panel's mission and the structure of the report

The panel was charged by the Engineering and Physical Sciences Research Council (EPSRC) and the Council for the Mathematical Sciences (CMS) with the task of making an independent assessment of the quality of UK research in mathematics from an international perspective. The formal terms of reference were:

> *To assess the standing and potential of mathematics research in UK universities and comparable institutions, in comparison with international work in the field. The Review should look at the width and quality of all aspects of research in mathematics and its applications, encompassing all of pure and applied mathematics, and statistics.*

Our reading of the panel's mission was to take a holistic view of the research achievements and status of UK mathematics, and of the potential for the future. The nature of this mission entailed that not all subject areas could be treated comprehensively. In the case of Statistics, for example, significant parts of the field relating to medical, biological or social science disciplines were not addressed by the panel.

The report is organized in three parts:

- The first part provides an overall review of the present standing of mathematical research in the UK. It addresses the considerable strength of British mathematics, and problems that have arisen in connection with research training.
- The second part addresses how to ensure the future vitality of British mathematics by attracting and nurturing the next generation of mathematical scientists. The panel draws particular attention to the increasing reliance of the UK on foreign-trained mathematical scientists, to the need for increased attention to career paths, and to the impact of the present form of the Research Assessment Exercise (RAE) on the way research is being conducted.
- The third part surveys the main challenges of the discipline from an international perspective, and identifies ways of meeting these challenges in the UK. Strengthening the linkages within mathematics, and between mathematics and other areas of science and technology, is critical to the future health of mathematics in the United Kingdom.

The main recommendations of the panel are listed in a condensed form at the end of the report.

Several appendices complete the report: Appendix A lists members of the panel; Appendix B gives a short description of the review process, and its implementation, while Appendix C lists documents put at the disposal of the panel and of the people whom the panel met during the sessions at the London Mathematical Society headquarters in London and during the site visits.

1. The Current State of the UK Mathematical Research Community

There are many deep intellectual links between pure and applied mathematics and statistics/probability. In making fundamental advances in these three areas of the mathematical sciences over the last 50 years, these links have been utilized in essential ways. For descriptive efficiency only, this report presents each category separately.

The contribution made by UK mathematicians to the advancement of mathematics has been recognized by many international awards, such as several Fields medals (ATIYAH, BAKER, BORCHERDS, DONALDSON, GOWERS, QUILLEN, ROTH and THOMPSON) and a special Silver Plaque awarded by the International Mathematical Union to the UK trained mathematician WILES. This puts the UK at par with France and the US for the highest international mathematical honour, which so far has favoured the pure aspects of the discipline.

1.1. Pure Mathematics

In pure mathematics the UK continues to be particularly strong in geometry and topology and its links with theoretical physics, as well as in number theory, all domains where the UK has a long tradition.

There are a number of other areas of strength, as well as some weaknesses as detailed below.

Algebra and representation theory

In algebra world-class work is being done in infinite group theory, finite groups, noncommutative algebra and representation theory.

All these areas lean on an old and strong tradition, and benefit from the presence in the UK of some of the best specialists who are active, inventive, and attract young mathematicians from all over the world.

Analysis

In analysis there are pockets of strength, in particular, in spectral theory, integrable systems, and numerical analysis. However, aside from several world-class senior researchers, the UK is suffering by not having a major representation in certain active areas of nonlinear PDEs amongst more junior staff, for example, parabolic-Ricci flow or nonlinear hyperbolic equations, from a hard analysis point of view. Moreover this weakness at Oxford and Cambridge affects other institutions in that these two institutions train many of the new Ph.D.s.

Combinatorics

Current research in combinatorics (another area that has become a major applied one) is done in a variety of institutions throughout the UK. The UK has a long tradition of fundamental work in graph theory, design theory and extremal combinatorics, with more recent significant activities in probabilistic combinatorics and combinatorial number theory. In view of the increasing demand for graduates in this area, especially from industrial, governmental, and academic employers, we are concerned that the supply of graduates in this area may not be adequate to fill the anticipated demand. In view of this demand, and the relative understaffing in this discipline in the UK, we feel a good argument can be made for building more capacity in combinatorics here.

In addition, we feel that valuable opportunities for this discipline are being missed because of the lack of strong ties to the theoretical computer science community, that are typically present in universities outside of the UK. Much of combinatorics now being developed world-wide has a strong algorithmic component, and as pointed out by the earlier International Review for Computer Science, this is a side of computer science which is not as strong in the U.K. as it should be.

Dynamical systems and ergodic theory

Dynamical systems and ergodic theory are flourishing research areas within the UK with over 200 researchers. As in other research areas, individuals trained outside the UK have been recruited in recent years, but much of the strength also derives from individuals trained in the UK over the past 30 years. The community participates in several European networks and interactions within the country seem to work well. There are strong interactions with other research areas such as fluid mechanics, numerical analysis, and mathematical biology, as well as attempts to develop stronger interactions with industry.

Geometry and Topology

UK mathematics has a long tradition in geometry and topology, and continues to be a world leader in these subjects. The mixing of algebraic and differential techniques to tackle all kinds of geometrical problems, continuing the path inaugurated by Sir William HODGE some 50 years ago, remains one of the distinctive features of geometry in the UK.

UK geometers have shown great flexibility in working across disciplines, combining algebraic analytic and topological methods, and having established strong links with mathematical physicists. There are a number of excellent research groups, some of them involving theoretical physicists having positions in mathematics departments.

Logic

In logic the UK is an international leader in model theory and has world-class researchers in proof and computability theories.

Number Theory

Number theory is a traditional strength in the UK, and has been for almost a century. Indeed, the UK has produced many world-class researchers and several Fields Medallists in this area. The areas of analytic number theory, algebraic number theory, diophantine equations and approximations, computational number theory and combinatorial number theory have been at different points in time particularly noteworthy.

The field evolves with developments in the subject and availability of leaders. One direction that has played a major role outside the UK, and which seems never to have developed a critical mass in the UK is the modern theory of automorphic forms (for example the “Langlands Programme”).

In contrast to earlier years, number theory is also a topic that now has major applications outside of mathematics and so the demand for graduates is bound to further increase. This is primarily due to the fact that the “digital revolution” taking place in the commercial sector relies on the digital representation of all types of data (text, voice, video, etc.) that must be stored, transmitted, protected and authenticated. Number theory has many of the tools needed to deal with these problems, and in fact, this discipline has now become one of the most applied subjects within mathematics (Hardy must be turning in his grave!). It is important that the UK continues to maintain this tradition of excellence.

Theoretical and Mathematical Physics

There is a very strong tradition in the UK in the more mathematical aspects of theoretical physics. The UK is world leading in the areas of theoretical particle physics, with special emphasis on string and supergravity theories, in general relativity and in cosmology. As such, it is ideally positioned for future developments on the interface of these subjects. It is a special feature of the UK that this research finds its place in mathematics departments, and so fosters connections with applied mathematics. There is a healthy flow of students from around the world to these UK research groups. It has proven possible to recruit and retain excellent junior faculty, especially using the mechanism of the long-term research fellowship, reversing a negative trend that manifested itself in the 1970s and 1980s. Lacking some visibility in the general area of mathematical physics is statistical physics (although it may be represented in physics departments) in contrast with the fact that probability theory is very strong and well represented.

1.2. Applied Mathematics

The UK has been an international leader in applied mathematics, especially in industrial mathematics and fluid mechanics with applications to astrophysics, biology, geophysics, geology, meteorology and oceanography. In particular nonlinear stability theory, triple-deck asymptotics, mechanics of animal motion, wave-interaction theory, shockboundary layer interactions, and aero-acoustics were invented here.

At present fluid mechanics, applied dynamical systems, numerical analysis and biofluids continue to thrive in the UK, but other areas of applied mathematics that are important world-wide are not so well represented. These include applications to materials science (development of microstructure in crystalline materials), quantum communications, quantum computing, and mathematical aspects of nanosciences in general.

Fluid mechanics

The need to understand turbulence, free-boundary problems and stratified and rotating fluids in fluid mechanics has provided the motivation for the development of many of the modern tools for the analysis of abstract evolution equations. In addition, there are intimate connections to modern dynamical systems theory, techniques of asymptotic analysis, bifurcation theory, computational methods, and more recently, to stochastic dynamics.

Analysis in fluid mechanics has been a major strength of the UK applied mathematics for at least 150 years, and has included figures such as KELVIN, RAYLEIGH, TAYLOR, LIGHTHILL, and BATCHELOR, and current research in this area is at the highest international level. A major success has been the understanding of nonlinear instabilities and their subsequent evolution. Another major component of research in this area is computational fluid dynamics, motivated by challenging problems in biology, geophysics, meteorology, oceanography, and magnetohydrodynamics. The level of research in computational fluid dynamics is generally of very high quality, research groups have critical mass in some institutions and are well-funded, and there appears to be an adequate stream of students into both analytical and computational fluid mechanics.

Numerical analysis and scientific computing

The UK was a pioneer in the earliest research on numerical analysis and the development of numerical methods, and that leadership continues today in several areas. Starting with J.H. WILKINSON in the 1950s, UK research in linear algebra has consistently been at an international level of excellence, addressing both fundamental theory such as stability and the production of robust numerical algorithms. In ordinary and partial differential equations, several UK groups are widely recognized for their work on multiscale and adaptive algorithms, stochastic differential equations, and preconditioning techniques. Two of the world's best known researchers in optimization (FLETCHER and POWELL) work in the UK, and the UK style of tightly combining theory and numerics remains a major influence in optimization.

Mathematical Biology

Mathematical biology is a relatively new addition to the mathematical community, but the application of mathematical tools has a long tradition in biology and medicine in the UK, as evidenced for instance in the Nobel-prize winning work of HODGKIN and HUXLEY 50 years ago and more recently, by the development of computational models of the entire heart.

Since mathematical biology is intrinsically an interdisciplinary subject, a great deal of modelling and analysis of biological phenomena is done in life science departments, often via collaboration between mathematicians, statisticians and scientists working in fields such as medicine, ecology and bioinformatics. It is a very vibrant area, a high percentage of the researchers are relatively young, and the research is well-funded and generally of very high quality. Mathematicians in the UK have been and continue to be world leaders in several areas of mathematical biology. These include

(i) pattern formation in developmental biology and more generally in chemically-reacting systems,
(ii) biological fluid dynamics, including blood flow and swimming microorganisms,
(iii) modelling of the electrical and contractile activity of the heart, as part of an international team,
(iv) tissue dynamics, including wound healing, tumour growth and angiogenesis,
(v) mathematical and theoretical ecology.

In addition, very good work is being done in the areas of theoretical immunology and in epidemiology. All of these areas involve a combination of mathematical modelling and analysis, and frequently of computational methods.

As will be discussed later, there is a clear need for a well-defined training programme in mathematical biology.

Financial Mathematics

Financial mathematics brings together methods of PDE, stochastic analysis, statistics and numerical analysis to develop methodologies for the investment-banking industry including optimal investment strategies, risk-management tools and pricing methodologies. The UK is among the world leaders with particular strength in the applications of stochastic analysis.

Industrial Mathematics

The UK has world-class activity in the organization of industrial mathematics. In particular, the annual “Study Group” that brings together industrial researchers and mathematicians has been a major success and is used a model for industrial mathematics in other countries.

1.3. Probability and Statistics

The subjects of probability and statistics confront the interface of mathematics and the empirically-based disciplines, particularly those dealing with complexity and uncertainty. They have been revolutionized by technological changes that have resulted in a huge change in scale of the empirical data and the computational power now widely available. As a result the focus in probability has turned to modelling more complex structures and systems and in statistics to new computational tools and methods of inference for complex data are being developed. The UK community has played a major role in these developments.

Probability

Research in both probability and the applications of probability is carried out at the highest international level in the UK. There is currently a thriving community that includes a number of international leaders and a remarkably strong and vigorous group of researchers. In part, this has been made possible by recruiting well-established and talented younger researchers from outside the UK.

The UK is among the world leaders in stochastic analysis including finite and infinite dimensional diffusions, stochastic partial differential equations, measure-valued processes, interacting particle systems, and stochastic numerical methods. The UK is also an international leader in probabilistic models originating in statistical physics and discrete probability including Markov chains, percolation, random walk in random media, random graphs and random matrices.

In the applications of probability the UK is an international leader in the analysis of probabilistic models of communications networks and internet traffic, coalescent methods in population genetics, randomized algorithms and particle methods for nonlinear filtering.

Statistics

Dating back to the inception of statistics (largely in the UK) more than a century ago, the UK has been a world leader in many areas of statistics, particularly in developing theory, methodology and modelling based on challenging problems in the sciences, as well as in business and industry. This glorious past still influences British statistics, notably in many important and innovative concrete applied statistical methodology and modelling initiatives, e.g., longitudinal studies in biostatistics, statistical extreme values in materials strength and climatology, ecology (animal abundance), agriculture, finance, and infectious disease epidemiology.

A newer development is the emphasis on Bayesian statistics, which has come to be a main research activity both as regards methodology and in actual data analysis, aided by important UK investments in statistical software, such as BUGS. The research in Markov Chain Monte Carlo algorithms and other areas of Bayesian statistics is leading in the world. In modern statistical genetics top class UK statisticians and probabilists have recently redirected their efforts to an extent that it is already clear that they will continue to play leading roles worldwide.

Statistics of shape and other areas of spatial statistics is a UK stronghold, with strong connections to applied probability.

Exploratory (data-analytic) methods have been developed and studied for many years, with important recent contributions being in the use of wavelets and in functional data analysis.

Statistics in the medical and social sciences was marginal to the brief of the panel. However, in both areas the UK is strong. It enjoys substantial excellence in the application of statistics to medical problems, and in the social sciences it is especially strong in sample survey methodology and econometrics.

1.4. Training Mathematicians at UK Universities

Undergraduate enrolments in mathematics are stable or just begin to show signs of decline, in contrast to those in some European countries. However, the Committee noted well-founded concerns with a decline in the quality of the mathematical training of incoming students and the number of teachers at the school level who do not have the appropriate level of mathematical qualifications. The former derives from the latter, at least in part, and new efforts are needed to reinstil a love of mathematics and the teaching of mathematics amongst school teachers. This is not, however, a peculiarly UK problem.

Mathematics graduates are highly valued by employers because of their analytical skills and problem solving abilities. Schools cannot compete financially for these people, who are in very high demand.

The Ph.D. programme currently produces around 200 doctoral students each year. The programme is of shorter duration and more narrowly focused than those in most other countries. As a result, Ph.D. graduates have difficulty competing for research fellowships and academic posts, and our perception is that, as a result, many go into non-academic careers in industry and the financial sector.

The system of three-year Ph.D.s can only work if there is excellent A-level education at the school level. Our perception is that the A-levels are weaker than they used to be. The result, then, is that this produces many students who cannot compete with graduates from abroad. One should say though that the system continues to function well for stellar students who can profit from the five year post-doctoral fellowships free of teaching duties.

It is projected that approximately 85 academic staff will reach retirement age each year over the next 10-15 years. In addition to this there will inevitably be other resignations. The general age profile on the faculty is reasonable but it is our perception that in some areas the situation is of potential concern. In particular, the area of statistics has outstanding scientists in the 50-and-above age group, but has experienced and experiences serious difficulty recruiting young talented statisticians. There is a serious concern that now the UK is not producing a sufficient number of Ph.D. graduates in statistics to satisfy the demand from industry and at the same time to maintain the level of excellence in universities. Anecdotal evidence suggests that in recent years a large proportion of new appointments at UK universities are made from outside the UK.

MSc programs in Statistics are of major benefit for both industrial and university sectors in the UK, but are perennially under threat. They should enjoy greater security and support. More generally the panel has heard repeatedly from employers of mathematicians that their analytical and problem solving skills are among their special attributes. In these times where the quality of personnel is the discriminant for the success of many industrial endeavours because of the decisive role played today by innovation and modelling, having access to a new generation of skilled employees properly trained in the latest developments of mathematical research in a broad sense has become an issue with far-reaching consequences for the prosperity of a nation.

1.5. Public Appreciation of Mathematics

Efforts are being made to raise the public awareness of mathematics but it is our perception that there does not exist widespread appreciation in society at large of the impact that mathematical research has made on the modern economy. It has never been easy to communicate mathematics to the broad public, compared to other scientific disciplines such as astronomy, biology or even physics. Nevertheless, it has become increasingly important that this be done, not only to be able to attract the best young talent into the field, but also to maintain the continuity of the funding necessary for the long-term health of the subject.

By and large, the average citizen is not aware of the enormous advances that have occurred in recent years, and how these advances are essential in modern day society (e.g., CAT scans, digital communication, error correction in DVDs, privacy and security for personal data, etc.), although recent occurrences of mathematics and mathematicians in the theatre and movies have raised their awareness and appreciation somewhat.

It is ironic that at the same time, the non-academic consumers of mathematics the Panel heard from are well aware of the value of graduates with advanced mathematical training and eagerly seek them out as employees and consultants. We recommend that increased efforts be made to explore ways for communicating the vitality, utility and beauty of mathematics to a much wider audience. The UK has a well-deserved reputation for first-class science programmes produced by the BBC, and efforts should be made to incorporate equally-imaginative approaches to contemporary mathematics on a regular basis.

2. Attracting and Nurturing the Next Generation

Because mathematics is very much a people-oriented discipline, establishing environments that nurture creative mathematics at all career stages in order to ensure that the next generation of mathematical leaders will be ready in time should be a top priority. In order to keep its leadership position in mathematics, the UK has to be internationally competitive. This means paying special attention to career paths, at the junior, mid-career and senior levels. These issues will become more and more important in the near future because of the high demand for qualified personnel that will result from the need to replace a full generation approaching retirement as the result of the simultaneous expansion of the higher education and research sectors in several countries. The industrial sector will also become a major player in the hiring of mathematicians, raising the financial stakes further.

2.1. Undergraduate Education

Remarkably, UK mathematics is not suffering from a decline in undergraduate students. Compared to the world, in particular the US and continental Europe, undergraduate enrolment levels are high and have remained more or less constant. There is also a widespread feeling that studying mathematics is a good path to a non-academic career, e.g. in finance or industry.

There are, however, some serious concerns that students are entering university less well prepared. Mathematical A-levels are not as rigorous as they used to be, and school mathematics teachers are less well trained in advanced mathematics. The community is paying and must continue to pay close attention to these developments. There are some very laudable initiatives in place to enhance the visibility of mathematics in schools (such as the Math Millennium project).

In order to draw the most talented students from across the country, it is important for excellent undergraduate education to be widely available. In view of the increasing importance of mathematical research to the economy at large, the UK cannot afford to have high-quality undergraduate education in mathematics limited to only a few universities.

Outside the main mathematics centres, more regional cooperation for undergraduates and M.Sc. students should be encouraged and monitored, allowing students in small departments access to a full spectrum of opportunities to learning mathematics.

2.2. Post-Graduate Education

One goal of UK mathematics should be to produce excellent Ph.D. graduates. Ph.D. programmes must be competitive at the international level if UK mathematics is to maintain its standing. This requires both attracting excellent students and providing them programmes of the highest calibre.

Because of the high quality of its faculty and education, the UK is still able to attract, besides a reasonable number of nationals, excellent non-UK Ph.D. students, many coming from Eastern Europe. There seem to be enough able candidates for positions in pure and applied mathematics, but definitely not enough candidates in statistics. The lack in statistics stems primarily from the existence of attractive alternatives in industry. It seems that EPSRC is aware of this and is providing resources for universities to offer enhanced stipends in an attempt to attract more students into statistics.

The three-year UK Ph.D. programme is short compared to Ph.D. programmes in other countries, and this has some negative aspects. Moreover, the current system is highly structured and inflexible. This structure produces Ph.D. students who are narrowly focused. New Ph.D.s from the UK usually have less breadth and experience than their peers from other countries.

In the European context, the Bologna agreement gives a framework for university education, setting a 3-2-3 year standard across Europe leading to the Ph.D. This eight year period from university entrance to the Ph.D. is intended to increase globally the international visibility and attractivity of European Ph.D.s. In order to keep UK mathematics graduates competitive and make the programme offered there as attractive as possible for non-UK students, mathematics education should try and follow this pattern. The panel was surprised that seemingly very little thought has yet been given to this perspective.

In contrast to these efforts, in the UK there are penalties and deterrents in place from the research councils if a Ph.D. study takes too long and students who require longer than three years to finish typically receive no, or limited, financial support. We heard accounts of host institutions being penalized if a supported student does not finish in three years. These pressures on completing the Ph.D. quickly influence mathematics research very negatively by discouraging ambitious research topics that offer the possibility of greater impact than ones that can predictably be completed in a three-year programme.

To attract and nurture the best Ph.D. students, the system needs more flexibility to respond to the multifarious needs of applied and theoretical parts of the Mathematical Sciences. One should give students the opportunity to develop more breadth in background, to change advisors or disciplines, and to learn about other disciplines. Our findings strongly confirm the recommendations of the Roberts report. We believe that the system should immediately institute flexibility in the time required for students to complete their Ph.D. studies, allowing some students to finish in three years while the majority of students take four years in order to gain a suitably broad and deep training. EPSRC argues that its Doctoral Training Accounts do offer this flexibility, but it not as yet had that effect, and the perception by departments seems to be still that of a rigid situation.

2.3. Post-Ph.D.

A large number of UK Ph.D. graduates find employment in business and industry. Here the three-year Ph.D. study is often seen as an advantage, although high quality M.Sc. programmes might actually produce equally desirable candidates for industry. However, in the UK as elsewhere, there are fewer research positions in industry or government labs compared to the past, with some exceptions such as GCHQ and certain statistics positions in industry and other research groups.

Since an academic career is the main option to pursue research in mathematics, it is crucial for UK-trained mathematicians to have postdoctoral opportunities that facilitate the transition to university faculty positions. Traditionally, universities hired new Ph.D. graduates as lecturers, expecting these graduates to have the experience and ability to develop independent research programmes. That is no longer the case. With the strong emphasis upon productivity of all “research active” faculty included in the RAE, an established research record is almost a sine qua non for obtaining a university faculty appointment. Therefore, non-faculty postdoctoral appointments have become the next step for most new Ph.D. graduates who seek academic careers.

Since the market for post-doc positions has become international, UK Ph.D.s may need several postdocs to have the experience needed to obtain an academic position in the UK today. There is an impression among the Ph.D. students and postdocs interviewed by the panel that there should be more postdoctoral positions within the UK that are not tied to specific projects. The three- and four-year EPSRC and Royal Society Fellowships give excellent opportunities to a select few individuals at this level, but many highly talented individuals must leave the country to find suitable positions. Statistics is an area, where, due to attractive opportunities in industry, EPSRC and other funders must provide more attractive stipends so as to offset the competition. There is a perception that there are not enough EPSRC fellowships available in statistics because applications in this area tend to be underrated due to the multidimensional skills that have to be evaluated positively; if true then EPSRC might consider ring-fencing a number of posts in statistics.

2.4. Academic Positions

Overall, the total number of faculty positions in the UK has increased and is close to meeting demand, certainly compared to the recent past. Among Ph.D. students and post-docs there was considerable optimism about the chances to get jobs, tempered by their expectation that they must establish a substantial research record to obtain a university faculty position. Our feedback from these individuals should be treated with caution, since we spoke only with a small sample of students and postdocs selected by our hosts.

Scholars pursue academic careers to teach and conduct research. However, in the UK junior faculty are burdened with many other activities that distract them from their teaching and research, for example, completing an increasing number of bureaucratic forms, committee service, and interviewing prospective undergraduate students. Compared to industry, lecturer positions are not well paid and carry a high administrative load in an increasingly constrained environment. A typical example of this is given by the insistence of the government that holders of higher education posts be certificated. The administrative burden is especially onerous on junior faculty in small departments that are struggling to provide a broad mathematics education, because these faculty must teach in many different fields while their training has been narrowly concentrated on their research area. In one case a student who had been in industry for a few years was shocked to find his brilliant adviser carrying out menial work that would never be done by any person with any responsibility in industry. Because the principal resource required for mathematics research is time, the increasing bureaucratic demands on university faculty have become a serious problem.

From a departmental perspective, departments are still able to hire good people because of the abundance of excellent candidates from non-UK countries. The UK, as many countries around the world, has had an influx of excellent mathematicians from Eastern Europe, but this cannot be counted on for the future because of the decline in the number of people trained in these countries and the slow but definite economic development happening there. At present the UK benefits from poor academic career options in some parts of the world, but it is expected that the competition for good candidates will be fierce during peak periods of retirements happening simultaneously in many developed countries, both in academia and in industry.

Advanced long-term fellowships such as the ones mentioned before are beneficial to academic careers. They have been awarded mainly to a small number of beginning faculty with especially active research programmes. However, the recipients should not be isolated totally from faculty responsibilities. It is important for these fellows to participate in the life of their departments and gain some form of teaching experience, more so than appears to happen at present.

Statistics, here to be considered as really distinct from Probability, is an area where the outlook is much more alarming than for pure and applied mathematics. There are few candidates from Eastern Europe, and departments have to compete directly with their US and Western Europe counterparts. More generally, the serious problems faced by statistics in the UK in ensuring adequate research staff and leaders for the future have to be addressed as such. A serious difficulty faced by academic statisticians is securing funding through the EPSRC grant competition. This difficulty was conveyed to the panel on several occasions during site visits. The perception among researchers in this area is that this difficulty is due to the fact that statistics proposals must be evaluated in a host of dimensions including mathematical contributions, relevance of the application, and novelty of the particular statistical method. This is in contrast to some other areas in which the dimension of mathematical contribution dominates. As a result statistics proposals may be more easily criticized. In particular, researchers felt that the evaluation and joint ranking of proposals from all areas of mathematics seems to place statistics proposals at a relative disadvantage. These perceptions should be addressed, and if found to be justified could be overcome for example by introducing an initiative in “contemporary statistics and stochastics”.

2.5. Some Special Issues Related to Careers

Dual-career couples

An increasingly large number of academics, especially younger ones, have dual-career families. This factor may need to be considered in recruitment and retention of top-quality new faculty. This is a delicate issue since the dividing line between good and bad practice on these matters is difficult to draw.

Mid-career

It is important to nurture the leaders of the future. There is an obvious lack of support mechanisms for mathematicians in mid-career (40s). More attention should be paid to retaining the best mid-career people and to offering improved working conditions for the most active ones in research. The UK is not competitive in that respect. There is in particular a danger that some leaders would move permanently to the US or elsewhere. More senior fellowships directed to mid-career individuals would be an option. Currently there is a severe dearth of mid-career statisticians as evidenced by the difficulties in filling chairs. This is a very unhealthy, if not outright perilous, situation.

British mathematics has taken up the task of modernising its internal organisation following the trends that one can witness in many of the leading countries in the field. This includes the setting up of institutes open to scientists coming from the whole world. The institutes hosting focused programmes developed over several months presently in operation, the Isaac Newton Institute for Mathematical Sciences in Cambridge and the International Centre for Mathematical Sciences in Edinburgh, contribute in an essential way to accelerating the development and applications of mathematical ideas. These institutes in the UK or their counterparts elsewhere in the world offer particularly favourable conditions for researchers, including university professors, to concentrate on their research projects, and to exchange ideas with other specialists at a high level. Making them more easily accessible to mid-career mathematicians for extended stays is certainly a good investment, and can play a big role in helping them cultivate their research strengths.

Senior

Although there are some mechanisms in place to recruit and retain world-leading mathematicians, the UK is not fully competitive on the global level in this respect. Some more thought should be put into this question because of the possible long-term adverse effects of lack of leadership. Reviving the research in some areas will very likely require constituting comprehensive teams around some leading and charismatic figures.

2.6. The Research Assessment Exercise (RAE)

One can not ignore the effects of the RAE on the culture and practice of UK mathematics research.

On the one hand the RAE has undoubtedly enhanced the importance given to research in departmental and university policies. Research excellence and good education go a long way together, and we commend the intent behind research assessments. With such a large number of Ph.D. granting institutions there is clearly a need to encourage institutions to focus on their strengths. The RAE focuses UK mathematics on research and rewards it, but unfortunately in somewhat biased ways, and this has clearly become an issue.

We give here the findings of the panel about the impact and the negative (unintended) consequences of the present RAE implementation.

Excellent sub-areas of mathematics have benefited because, through the RAE, they have been brought to the attention of university administrations and research communities outside mathematics, a new situation which has resulted in the availability of more resources. The RAE has also enhanced the visibility of mathematics research within the UK.

In many respects though, the impact of the RAE goes much beyond what it was supposed to achieve, and from this point of view the impact on mathematical research seems to be more negative than in other fields because of the smaller critical size of teams required to perform quality research.

Aspects of the RAE are the heavy administrative burden on the departments for the preparation of the RAE, discouragement of risk-taking and the encouragement of incremental research (as one junior faculty member commented, “I need an RAE paper”). There is a major risk that this could hamper the future excellence of UK mathematics research. A better balance between the need for periodic assessment and the cost of doing it must be found.

The RAE does also lead to personnel decisions that can create tension within departments, for example, in hiring people at the right time to contribute to the RAE submissions, and in deciding which faculty will be included or excluded in a submission. The latter process is especially counter to collegiality and may reduce productivity by certain members of the faculty who have been classified as inactive in research, due to the excessive weight given to formal evidence of that.

The large discontinuity in resources available to universities based on their RAE score has a negative impact on mathematical research by fostering a narrow spirit of competition between departments. Mathematics is a discipline where small teams organizing themselves in networks can achieve an excellent level of exchange and create a highly stimulating environment for advanced students and young researchers. The UK cannot afford to have its high quality research concentrated in too few leading departments that are competitive at an international level. Cooperative work between distinct universities in well defined research areas of mathematics must be encouraged. From this point of view enhancing the possibility of finding support for regular meetings between distributed teams seems particularly appropriate, in that it can counterbalance the effect induced by the spirit of strict competition between universities that the RAE encourages.

3. Opportunities and Challenges

Looking into the future is of course the most important, and riskiest, part of this report. There are two parts to our recommendations.

First, we make brief suggestions about areas within mathematics which appear to us timely as priority areas for additional investments within the UK, taking up the list of areas discussed in Section 1. Of course, our judgment is coloured by our experiences and interests, and our recommendations necessarily are influenced by these. Moreover, it is likely that unanticipated scientific events will add topics that we did not identify, and reduce the impact we expected from some new developments.

Second, we identify opportunities and challenges spanning the breadth of mathematics (and often beyond to other disciplines) that are structural in nature. In most instances, we do not make specific recommendations about these since further discussion is warranted before mechanisms are chosen to address these issues.

3.1. Domains where UK mathematics is world leader

Geometry/Topology/Theoretical Physics

The UK is undoubtedly a world leader in Geometry/Topology/Mathematical Physics. Its strength is based on outstanding individuals leading significant groups spread over a dozen universities, working on a broad range of subjects and interacting with other areas (especially with theoretical physics). This group contains excellent young geometers, joined by some mathematicians from the former Soviet Union. The outstanding quality of differential algebraic geometry and global analysis, algebraic geometry, mathematical physics, topology is appreciated and recognized worldwide. This exceptional role of UK mathematics must be maintained and supported in the future. There are excellent opportunities in this area to strengthen the contacts between geometers and theoretical physicists, perhaps through joint appointments. The Theoretical Physics groups – as far as they are in Math Departments – are of excellent level, and fully internationally competitive.

It should be noted though that the linkage of geometrically minded specialists in Global Analysis with their counterparts in PDEs has not yet been made at the proper level, mainly because this latter domain is so thinly represented in the UK.

Number Theory

The UK continues its long tradition of leadership in number theory. Not only have UK based or trained mathematicians been leading the international explosion of arithmetic algebraic geometry and its spectacular achievements, but the scope of UK activity in number theory is large and international. Work on the global Langlands programme is one area of number theory that could be developed and expanded further, since it has become one of the main sources of motivation for research in arithmetic, geometry, number theory and representation theory internationally.

Probability and Statistics

The UK is a world leader in key areas of Probability and Statistics. For example, in probability the UK has been a world leader in developing the interface between major theoretical advances in stochastic analysis and related topics, and applications to fields as diverse as theoretical physics, finance, biology and engineering.

Because of the difficulties in attracting enough young people to academic positions in the field, statistics is dealt with later in this section, and specific steps suggested.

3.2. Domains where Maintaining the Leadership Requires Special Actions

Fluid Mechanics and the emergence of Computational Fluid Dynamics

The UK is a world leader in Fluid Mechanics. Its strength is deep and broad with emphasis in applications to biology, geophysics, meteorology, oceanography, and magneto-hydrodynamics. There are excellent faculty at all levels of seniority, and a fine stream of potential post-graduate students.

Research in Fluid Mechanics has been transformed in recent years because of the new dimension taken by massive computer simulation, to the point of giving rise to a new field, Computational Fluid Dynamics (CFD).

While research in Fluid Mechanics remains at the highest level, its long-term future must be secured by long-term investments in the new frontier that represents a thoroughly computational approach to several areas where modelling is now accessible but requires massive computations. In the context of major developments worldwide in CFD, having access to such resources is likely to become a condition for future successful developments.

Mathematical Biology and its new dimension

The UK also holds a leading position in Mathematical Biology. This should serve as a pioneering basis for a field which has taken a new dimension and now encompasses mathematical aspects of Genomics, Bioinformatics, and Ecology, besides more traditional Mathematical Biology. It is overall concerned with the study of complex systems, and offers many challenges.

There is an enormous opportunity for mathematicians in the development of new mathematical tools for understanding complex systems. The organization of biological systems is hierarchical and complex, and understanding how these structures function is a fundamental challenge for both biology and mathematics. For instance, sequencing of the human genome has not revealed how molecular-level phenomena produce the functioning of tissues, organs and organisms. Mathematics also plays an important role in our efforts to understand the functioning of ecosystems and in limiting the spread of animal and human diseases, as is evident in the recent efforts made to deal with the outbreak of foot and mouth disease in Britain.

The new developments have opened an era in which ad hoc training on an individual basis no longer suffices. There is a critical need for a larger mathematical component in the training of life scientists.

However, at present there is little agreement, either within the UK or the remainder of the world, as to what the appropriate mathematical and scientific components of training in mathematical biology are. The area is intrinsically interdisciplinary, so individuals trained in both mathematics and biological applications have not had a well defined niche within universities. Mathematics departments need to be one part of solving this problem. The UK is doing at least as well as other countries in this regard, but sustained efforts are needed. The EPSRC-funded Doctoral Training Centres at the life sciences interface provide an excellent opportunity for experimentation in this area, and may provide the first step in the development of a well-defined educational pathway.

Numerical Analysis and Computational Science

Computing has had a profound impact on research across the sciences. Computation is widely seen as a new partner that has joined theory and experiment as a strategy for doing science. Many countries have made large investments in research in scientific computing and computational science, often located in interdisciplinary centres and government laboratories.

The research in this domain both addresses the challenges of specific research areas and the need for common tools and methods in areas such as parallel computation. Even the research on this "core" of computational science is highly interdisciplinary, involving important contributions from numerical analysis. To cite only a single example, improvements in the algorithms for iterative and sparse methods in linear algebra computations have contributed at least as much as improvements in the speed of computers to the numerical solution of partial differential equations.

The United Kingdom was an early leader in such efforts, with work concentrated in several important laboratories such as Harwell and the National Physical Laboratory. Numerical analysis research in the UK is connected to this type of computational science, but the scale of activity is small compared to its scientific importance.

The EPSRC seems to have taken up the task of launching major projects in computational science, e.g. "Computation and numerical analysis for multiscale and multiphysics modelling". This is an important turning point, but to remain at the level presently achieved in Numerical Analysis much more will be required, and the proper linkages with applied areas, and industrial labs in some cases, will be indispensable.

Moreover, there is presently little activity towards the development of algorithms that connect numerical analysis and computer science, and in the context of major developments in computational science this may appear as a major issue that needs to be addressed.

Statistics

The UK has made major contributions to modelling, spatial statistics and Bayesian analysis, to name just a few.

Most of the important statistical theory and methodology has originally derived from dedicated analysis of important substantive problems, and this has been particularly true in the UK. A versatile and up-to-date background in mathematics, probability and mathematical statistics has however always been especially valuable for obtaining authoritative results.

Many recent application areas concern data sets so large and/or complex that professional skills in computer science are also required. On this basis the obvious advice to the future development of British statistics is to keep the best of the traditional dedication to practical importance while in many areas, including biology, finance, finding ways of attracting mathematically strong candidates to a training programme (probably extended beyond the present three-year Ph.D.) enhanced in the direction of broad background in mathematics, probability, mathematical statistics and computing.

Nevertheless, the future holds very significant dangers for UK Statistics, which is failing to produce enough young research leaders to fill even currently available Chairs, let alone those which will become vacant in the near future. The seriousness of this problem is exacerbated by the rapidly escalating demand for Statistics and statisticians outside the university sector.

3.3. Domains where UK mathematics is excellent

Algebra

Algebra is excellent in UK, with particular strength and influence in group theory, noncommutative algebra and representation theory. It continues to attract young, bright mathematicians from overseas.

Nonetheless, the coverage of areas in algebra is far from uniform. We cite two examples. First, "high technology" algebra such as in the works of Kontsevich or Voevoedsky is hardly represented in the UK as it is in the US and France. Second, research on topics such as Lusztig's conjectures, as well as conjectures in finite group representation theory, would benefit from better linkages with other areas such as algebraic geometry, topology, and Lie combinatorics. We speculate that a reason for these omissions might be found in the narrowness of the Ph.D. education in the UK. The general trend in contemporary algebra of enhanced communication with various areas of mathematics is difficult to incorporate into a program with a three-year undergraduate degree followed by a three-year Ph.D. program. Efforts to compensate for such narrowness seem warranted, for example by lengthening the education of algebraists, by hiring leading algebraists, by organizing programmes for regional and interregional cooperation, by emphasizing high level instructional workshops, etc.

Combinatorics

Most areas of combinatorics are present in the UK, many of them at an excellent, sometimes outstanding level. Combinatorics is present or implicit, at a very high level, in many "other" subjects such as group or representation theories. Originality, openness and some spectacular achievements characterize UK combinatorics. Demographic trends are nevertheless a cause for concern, since many of the current leaders are approaching retirement age.

In view of the increasing demand for graduates in this area, especially from industrial and governmental employers, it is worrisome that the supply of graduates in this area may not be adequate to fill the anticipated demand. This is also true for academic positions as well. In addition, it seems that valuable opportunities for this discipline are being missed because of the lack of strong ties to the theoretical computer science community (as compared with the situation in many other countries). A lot of combinatorics now being developed has a strong algorithmic component and as pointed out by the earlier International Review Panel for Computer Science, this is a side of computer science which is not as strong in the UK as it should be.

Dynamical Systems and Ergodic Theory

International leadership in the subject was centred primarily in the United States and Russia thirty years ago, with a flowering of the subject stemming from the vision of “giants” like Vladimir ARNOLD, Jürgen MOSER, Yasha SINAI and Stephen SMALE. Further development has been diverse with broader leadership. British mathematicians working in the subject have been part of many of the subsequent advances, but only occasionally have they provided the key insights that have initiated new parts of the subject.

Applied aspects of dynamics have been emphasized in the UK during the past fifteen years, and the community has reached out vigorously to other areas of science. This is a strength that can be further enhanced as the UK mathematics community seeks to maintain its role in training individuals for careers in business and industry.

3.4. Domains that are thinly represented in the UK

Discrete Algorithms and Computational Complexity

Computer science arose from a confluence of hardware, the physical development of computers and the design of their architectures, and software, the languages, operating systems and algorithms that are used to accomplish tasks on the physical devices.

The software parts of computer science were largely initiated by mathematicians, and mathematics continues to play a fundamental role in the development of algorithms. This has led to incredible growth in research on discrete mathematics and computer algorithms in other countries. This research area is underrepresented in the UK, both in mathematics and computer science departments. (See the remarks on theoretical computer science in the recent International Review of computer science.)
We recommend that the mathematics community takes a proactive role in addressing this issue, both by increasing research in the area and by establishing closer relationships with computer science departments. This also means that clear training paths at this interface have to be created, if possible with some visibility.

Analysis and Partial Differential Equations

Our committee finds that British research efforts in core areas of modern analysis and related areas of partial differential equations are sparse and sporadic in their coverage of international developments of these subjects. There are a few outstanding individuals in these fields, but the subject has not been a popular one among students.

Anecdotal information from our meetings with young mathematicians gives the impression that analysis is regarded as a difficult subject for a Ph.D., and that the stress involved with completing a dissertation in three years prompts few to choose the area. We believe that the subject will continue to be an essential component of mathematics and that it is important that it be well represented within the British mathematical community. Some determined action in this direction has to be taken, because this can undermine ability to develop some crucial areas of applied mathematics in the long run.

Materials Science

The bulk of materials science research concerns the fabrication and characterization of materials. Mathematics can contribute heavily to the modelling of processes, for example to the description of crystalline microstructure in hard materials. The microstructure determines the mechanical, electrical and thermal properties of the materials. At an international level, there is intense interest in nano- and micro-scale arenas with regard to the formation of protective coatings and the manipulation of quantum dots for quantum computing. Such studies utilize partial differential equations with anisotropic coefficients for free boundary problems and molecular dynamics simulations of ultra-thin layers and self-assembly.

There are individuals in the UK who contribute to this field, but it has not been an area of major investment. This is recognized in the recent international review of material sciences. The associated area of macro- and microscopic solutions of small scale systems is an area of intense mathematical interest internationally. The International Review Panel of material sciences describes the UK as "not a player" in the key area of multi-scale modelling.

3.5. Some structural issues

How to fill gaps

The emphasis on frequent review and evaluation of all research activities in British universities has led to substantial increases in the average quality of research at some universities, probably at the expense of others. However, the frequent assessment also makes it difficult for departments to establish activities in interdisciplinary areas or areas that are poorly represented. Developing new activities with critical mass is a difficult process that requires substantial investments and long term commitments. The same applies to revitalization and further development of underrepresented areas.

There appears to be broad recognition of these problems within the British academic community. However, mathematicians may be spontaneously more conservative than specialists of other disciplines in addressing them. Moreover, it is difficult to see how decentralized decision making by individual universities will lead to coherent national responses to the needs that we have identified.

For these reasons, national mechanisms to attain adequate coverage of different research areas and to exploit exciting new developments that reach beyond areas well established in the UK appear to be needed. It should be stressed again here that Mathematics is by far one of the least expensive sciences to develop, with a very high potential return, provided the investment is kept long enough and the appropriate links with the community built.

Regional and interregional cooperation

Regional cooperation among universities benefits large centres as well as being absolutely vital for small centres, where researchers may be isolated. Postgraduate students in small departments that are highly concentrated in a few areas need mechanisms to give them a broader view of mathematics than the one that may be accessible in their own departments. Seminars, meetings and workshops are a key source of information about what is happening at the international level, despite the growing importance of electronic communications.

Initiatives which put researchers from different centres in contact on a regular basis (like the BLOC – Bristol Leicester Oxford – Colloquium, or the Welsh initiative presently under consideration) should be helped and encouraged. As mentioned earlier, institutes play a very positive role in this direction. The London Mathematical Society, through activities such as the LMS Durham Symposia, and the Royal Statistical Society provide mechanisms for bringing mathematicians and statisticians from around the UK together. These efforts must be continued, and if possible amplified. Unfortunately, the perception is that the general tendency among Research Councils does not push in the same direction; true or not, more should be done. For mathematical research small can be beautiful. One has to create light structures to help fight isolation, and a number of initiatives should flourish.

Investing in people

As we have emphasized throughout this report, mathematics is a discipline driven by the creative insight of individuals. Outside of areas like computational science and data mining of enormous data sets, mathematics does not depend upon large investments in technology or the formation of large teams of researchers tackling a single problem. Consequently, particular attention must be given to creating professional environments that nurture creativity. Currently, the UK gives special opportunities such as five year fellowships to a few carefully selected individuals while increasing bureaucratic demands diminish opportunities for everyone else. Moreover, the systemic pressures of an enforced three year – Ph.D. and the gaps in career opportunities at the postdoctoral level create difficult obstacles for all but the select chosen few in developing strong academic careers. This is not a healthy situation.

In contrast to these problems, we reiterate how impressed we were at the involvement of mathematics departments in training substantial numbers of undergraduate and graduate students for careers in business, finance and industry. We recommend that the UK mathematics community take steps to further encourage students to choose mathematics as a suitable discipline to prepare them for non-academic careers.

The Panel's Main Findings and Recommendations

1. The mathematical sciences are increasingly playing a more central role in the development of advanced modern societies because frequently advanced technology relies on sophisticated mathematical content. Mathematical research has to be considered from this angle, and therefore must be developed ambitiously. It must be properly connected to advanced training to meet the needs for properly trained personnel for the future, in industry and in academia.

2. The panel looked at pure maths, applied maths and statistics across the country. There are a number of areas where the UK mathematics is a world leader, and several of excellent level.

3. Mathematical research is a people intensive activity with competition at an international scale. From that point of view the panel sees some dangers:
- *six years from entering university to getting a Ph.D. almost inevitably induces a narrow training; bringing the British system in line with the perspective offered by the Bologna agreement should be considered;*
- *difficulties in recruitment and retention of qualified personnel have appeared and are likely to worsen;*
- *the age distribution of some disciplines such as statistics is very unbalanced, and may heavily hamper future developments;*
- *the UK cannot afford to concentrate its advanced training in mathematics, which has to be nurtured by the most up-to-date research, in a small number of highly competitive universities.*

4. Worldwide challenges for the future of research in the mathematical sciences are:
- *to advance the core of mathematics,*
- *to develop linkages:*
  - *within mathematics,*
  - *between mathematics and other disciplines,*
  - *between mathematics and industry.*

5. On the one hand, the UK is meeting these challenges thanks to:
- *its rich tradition and talented people,*
- *areas with very strong linkages to applications (fluid mechanics, financial mathematics, industrial mathematics, mathematical biology, statistics),*
- *excellent international connections.*

6. There are concerns though for the future of research in the mathematical sciences in the UK:
- *the leadership for the future in statistics and some other areas is not at all ensured;*
- *the loss of a small number of key individuals can catastrophically affect the standing of areas of excellence;*
- *the interface with computer science is not developed at the appropriate level;*
- *UK graduates have presently difficult research career paths, and this may affect in the long run their willingness to work in the area when they are needed in larger numbers.*

7. Some impediments to the sound development of research have been detected and must be dealt with:
- *too much paperwork related to Research Assessment Exercise (RAE) and the audits undertaken by the Quality Assessment Agency is wasting the precious time of research fellows and academic personnel;*
- *the RAE has unintended consequences, e.g. the discouraging of risky research, the reduction in the diversity and the cohesiveness in the research community to name the two major ones for the area of mathematical sciences.*

Appendix A: The International Review Panel

Jean-Pierre BOURGUIGNON, *Chairman*
Directeur de recherche at the Centre National de la Recherche Scientifique, Director
Institut des Hautes Études Scientifiques, Le Bois-Marie, 35, route de Chartres,
F-91440, Bures-sur-Yvette, France.
+33 1 60 92 66 70, jpb@ihes.fr

Michel BROUÉ
Professor at the University Paris VII, Director
Institut Henri Poincaré, 11 rue Pierre-et-Marie-Curie, F-75231 Paris Cedex 05, France.
+33 1 44 27 64 18, broue@ihp.jussieu.fr

Stephen DAVIS
Professor
Department of Engineering Sciences and Applied Mathematics, Northwestern University, Evanston, IL 60208, USA.
+1 (847) 491-5397, sdavis@northwestern.edu

Don DAWSON
Professor
School of Mathematics and Statistics, Carleton University, 1125 Colonel By Drive, Room 4302 Herzberg Building, Ottawa, Ontario, Canada K1S 5B6.
+1 (603) 520-2600 ext 2147, ddawson@math.carleton.ca

Robbert DIJKGRAAF
Professor
Korteweg-de Vries Institute for Mathematics, University of Amsterdam,
Plantage Muidergracht 24, 1018 TV Amsterdam.
+31 20-525 5745, rhd@science.uva.nl

Ron GRAHAM
Director
Cal-(IT)², University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0405, USA.
+1 (858) 534-2086, graham@ucsd.edu

John GUCKENHEIMER
Professor
Mathematics Department, Malott Hall, Cornell University, Ithaca, NY 14853, USA.
+1 (607) 255 8290, gucken@cam.cornell.edu

Peter HALL, FRS
Professor
Centre for Mathematics and its Applications, Australian National University, Canberra, ACT 0200, Australia.
+61 2 6249-3474, peter.hall@anu.edu.au

Niels KEIDING
Professor
Department of Biostatistics, University of Copenhagen, Blegdamsvej 3,
DK-2200 Copenhagen N, Denmark.
+45 35 32 79 03, N.Keiding@biostat.ku.dk

Susan MURPHY
Professor
439 West Hall, 550 E. Univ, University of Michigan, Ann Arbor, MI 48109-1092, USA.
+1 (734) 647-3684, samurphy@umich.edu

Hans OTHMER
Professor
School of Mathematics, University of Minnesota, Minneapolis, MN 55455, USA.
+1 (612) 624-8325, othmer@math.umn.edu.

Peter SARNAK, FRS
Professor
Department of Mathematics, 906 Fine Hall, Washington Road, Princeton, NJ 08544-1000, USA.
+1 (609)-258-4229, sarnak@math.princeton.edu

Margaret WRIGHT
Professor and Chair
Computer Science Department, Courant Institute of Mathematical Sciences, Warren Weaver Hall,
Room 412, 251 Mercer Street, New York, NY 10012, USA.
+1 (212) 998-3056, mhw@cs.nyu.edu

Appendix B: The Review Process

The Review was overseen by a Steering Group comprising representatives of the EPSRC and the key learned and professional bodies (IMA, LMS and RSS), as follows.

Chair: Professor Martin TAYLOR
Nominees of
Institute of Mathematics and its Applications: Professor Tim PEDLEY,
London Mathematical Society: Professor Nigel HITCHIN,
Royal Statistical Society: Professor Bernard SILVERMAN,
Chief Executive of EPSRC: Professor John O'REILLY,
EPSRC Observer: Dr Annette BRAMLEY,
CMS Observer: Mr Peter COOPER,
Scientific Secretary: Dr Stephen HUGGETT.

The Steering Group was responsible for defining the structure of the International Panel and for the selection of its members. In consultation with the Chair of the international Panel it decided on the data to be collected and collated for the Panel, the choice and nature of the regional meetings, and the planning of the programme for the Review week. It was supported by the Scientific Secretary, Stephen HUGGETT, who organised the arrangements for the Review and assembled the materials for the Panel.

The International Review Panel comprised 13 (including the Chair) mathematicians and statisticians, all based outside the UK. (Membership is given in Appendix A.)

In advance of the Review week the Panel members were provided with a Data Document (containing statistics on mathematics research and mathematicians in the UK, outlines of UK research structure and funding mechanisms, etc.) and a set of Landscape Documents (providing a brief overview of the research being done in various fields). More details of these documents are given in Appendix C. These documents have been produced through a collective effort which was supervised and monitored by Stephen HUGGETT.

The main part of the Review took place over the one-week period 1–6 December 2004, and comprised two main elements:

a) briefings to supplement the data the Panel had received;
b) visits (by sub-groups of the International Panel) to eight venues across the UK where they received presentations on exciting areas of mathematics research being carried out in universities in the area.

The Panel members also had meetings with both senior and junior academics, postgraduate and postdoctoral students.

Each regional meeting was hosted at a university, and the programme planned by a number of universities in the region, as follows:

- Venue: Bristol University, organised with the universities of Bath, Cardiff, Exeter, Portsmouth, Southampton and Swansea;
- Venue: Cambridge University (including the Isaac Newton Institute);
- Venue: Durham University, organised with the universities of Leeds, Newcastle and York;

- Venue: Edinburgh University, organised with the universities of Aberdeen; Dundee, Glasgow, Heriot-Watt, St Andrews and Strathclyde;
- Venue: Imperial College, organised with the universities of Brunel, Holloway, Kent, Kings, Queen Mary, Reading, Surrey, Sussex and UCL;
- Venue: Manchester University, organised with the universities of Keele, Lancaster, Liverpool, Salford, Sheffield and UMIST;
- Venue: Oxford University;
- Venue: Warwick University, organised with the universities of Birmingham, Leicester, Nottingham and UEA.

The International Panel met as a whole for the final two days to collate its results and to begin to formulate its final report. On the final day, the Panel presented its preliminary conclusions and recommendations to the Steering Group and representatives of the sponsoring bodies.

The timetable for the Review during the week in the UK is given below for information.

Monday 1 December

afternoon: Arrival
evening: Reception and dinner with the Steering Group. Briefing session

Tuesday 2 December

morning: Briefing session
early afternoon: Planning for the sub-groups visits to the regional meetings
late afternoon: Panel (as four sub-groups) travels to first venues
evening: Dinner with senior academics at first venue

Wednesday 3 December

morning and early afternoon: Presentations on mathematics research in each of the four regions, discussions with academics and students
late afternoon: Sub-groups leave first venues and travel to second venues
evening: Dinner with senior academics at second venue

Thursday 4 December

morning and early afternoon: Presentations on mathematics research in each of the four regions, discussions with academics and students
late afternoon: Sub-groups leave second venue and return to London

Friday 5 December

morning: Briefing session
afternoon/evening: Panel private discussions and development of report

Saturday 6 December

all day: Report development and drafting
late afternoon: Presentation (oral) of conclusions and recommendations to Steering Group, participating organisations, etc.
evening: Farewell dinner for Panel and Steering Group.

Appendix C: Information provided to the International Review Panel

The Panel was provided with three sets of data:

(a) a Data Document sent to the Panel in advance of its visit, which was supplemented by additional data during the Review Week itself;

(b) a set of Landscape documents, providing a brief overview of the research being done in various fields;

(c) oral briefings during the Review Week on various aspects of the way the UK funds and supports mathematics.

(a)(i) Contents of the Data Document

SECTION 1: PEOPLE IN MATHEMATICS

1.1 Numbers of university staff
1.2 Age profile of the mathematics community
1.3 Women in the mathematics community
1.4 Ethnicity profile of the mathematics community
1.5 Undergraduate inflow
1.6 Graduate academic destinations
1.7 Individual fellowships and studentships
1.8 Mathematics departments

SECTION 2: FUNDING OF MATHEMATICS

2.1 Funding Council research allocations
2.2 Income to departments
2.3 Outline description of the Research Assessment Exercise (RAE)
2.4 Summary reports of the three Mathematics RAE Panels

SECTION 3: ORGANISATION OF AND POLICIES FOR SCIENCE

3.1 Funding of research in the UK
3.2 Spending Review 2002
3.3 EPSRC Mathematics Programme
3.4 Report of Sir Gareth Roberts Review “SET for success”
3.5 United Kingdom involvement in European programmes

SECTION 4: FURTHER MATTERS

4.1 The Royal Society
4.2 Faraday Partnerships and the Smith Institute
4.3 Bibliometrics
4.4 Mathematics and Statistics Departments, Centres and Institutes
4.5 Acronyms

(a)(ii) Additional Data provided during the Review Week

1. RAE results by Department and Unit of Assessment.
2. Proportion of Maths and Statistics researchers holding EPSRC grants.
3. Total number of mathematics undergraduates.
4. Threats to University mathematics departments.
5. Grants awarded in mathematics and statistics by other Research Councils.
6. Mathematics and e- Science.

(b) Landscapes Documents

Algebra
Analysis
Combinatorics
Dynamics
Financial Mathematics
General Relativity and Cosmology
Geometry
High Energy and Quantum Physics
Industrial Mathematics
Logic
Mathematical Biology
Number Theory
Numerical Analysis
Probability
Statistics
Theoretical Mechanics
Topology

(c) Briefings given during the Review Week

The Research Councils, given by Professor John O'REILLY, Chief Executive of EPSRC.
The Research Assessment Exercise, given by Professor Nigel HITCHIN, Savilian Professor of Geometry, University of Oxford.
Learned societies support for research, given by Professor Frances KIRWAN, President of the London Mathematical Society.
The Royal Society, given by Lord MAY of Oxford, President of the Royal Society.
The Research Institutes, given by Professor John TOLAND, Director of the ICMS, and Sir John KINGMAN, Director of the Isaac Newton Institute.
Mathematics and GCHQ, given by Dr Cliff COCKS, Chief Mathematician, GCHQ.
Users' forum, chaired by Professor John MCWHIRTER, Senior Fellow, Qinetiq.
Participants: Dr Robert LEESE (Smith Institute, Oxford), Dr Mark ROBSON (Bank of England), Dr Ian ROULSTONE (Meteorological Office, Reading), Dr Ben JEFFRYES (Schlumberger, Cambridge), Dr David STANDINGFORD (BAe Systems, Filton).

Council for the Mathematical Sciences
De Morgan House, 57–58 Russell Square, London, WC1B 4HS.
Tel: +44 (0)20 7637 3686. Web: www.cms.ac.uk

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